Improving spherical harmonic transform for the fine resolution global atmospheric spectral modeling

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Motivation

- Recent trend to have fine resolution and/or non-hydrostatic global models
- NCEP GSM is testing T878, T1148, and T1534
- The future to have T2000 may not be too far
- Current GFS is based on spectral computation, the question is “ Can GFS run very high resolution? “ or “Is spectral transform valid for very high resolution?”

Spherical harmonic spectral transform

Grid to spectral

$$q^m(\mu_j) = \frac{1}{I} \sum_{i=1}^{I} q(\lambda_i, \varphi_j) e^{-im\lambda_i}, \qquad (1) \qquad \text{FFT}$$

$$q_n^m = \sum_{j=\pm 1}^{\pm J} q^m(\mu_j) P_n^m(\mu_j) w_j, \qquad (2) \qquad \text{LT}$$

Spectral to grid

$$q^m(\mu_j) = \sum_{n=m}^{N} q_n^m P_n^m(\mu_j), \qquad (3) \qquad \text{LT}$$

$$q(\lambda_i, \varphi_j) = \sum_{m=-N}^{N} q^m(\mu_j) e^{im\lambda_i}. \qquad (4) \qquad \text{FFT}$$

I >= 3N + 1 or I >= 2N + 1

Approach

- How to do spectral transform in a fine resolution global spectral modeling?
  - Spectral transform is time consuming but it has its own advantages
  - We have reduced spectral transform (Juang 2004)
  - FFT is okay with fine resolution as we know
  - Is Legendre transform okay?

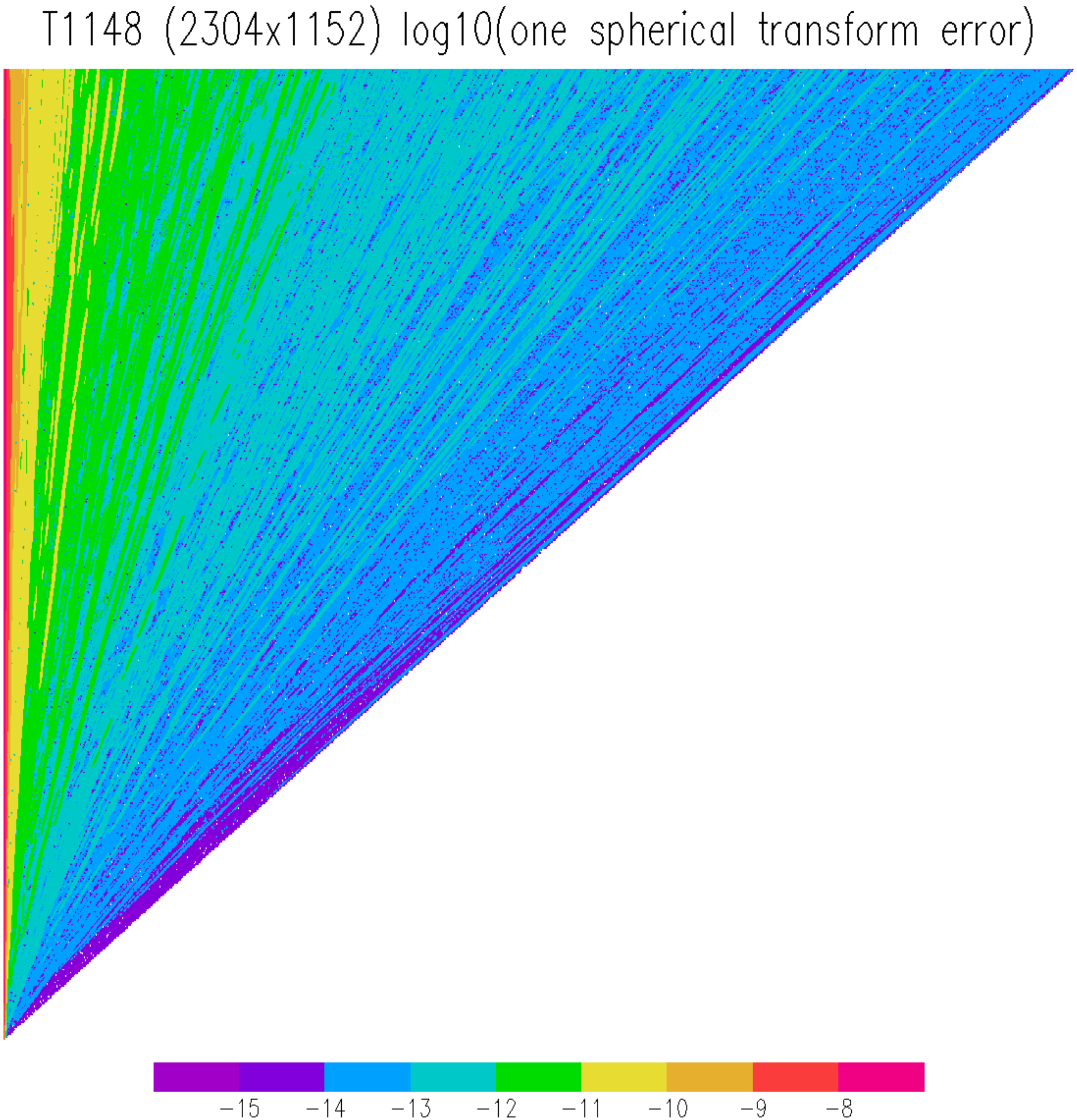
Method

- Use NCEP sp lib to test out spectral transform, since sp lib is used for GFS, such as chgres, GSI, postprocessor, etc
- Experimental method
  - Preset value of unit for spectral coefficient in real part and zero for imaginary part as (C)
  - Do spectral to grid transform as G
  - Then grid to spectral transform to final coefficient (D)
  - The difference between preset (C) and final values (D) after one spherical transform

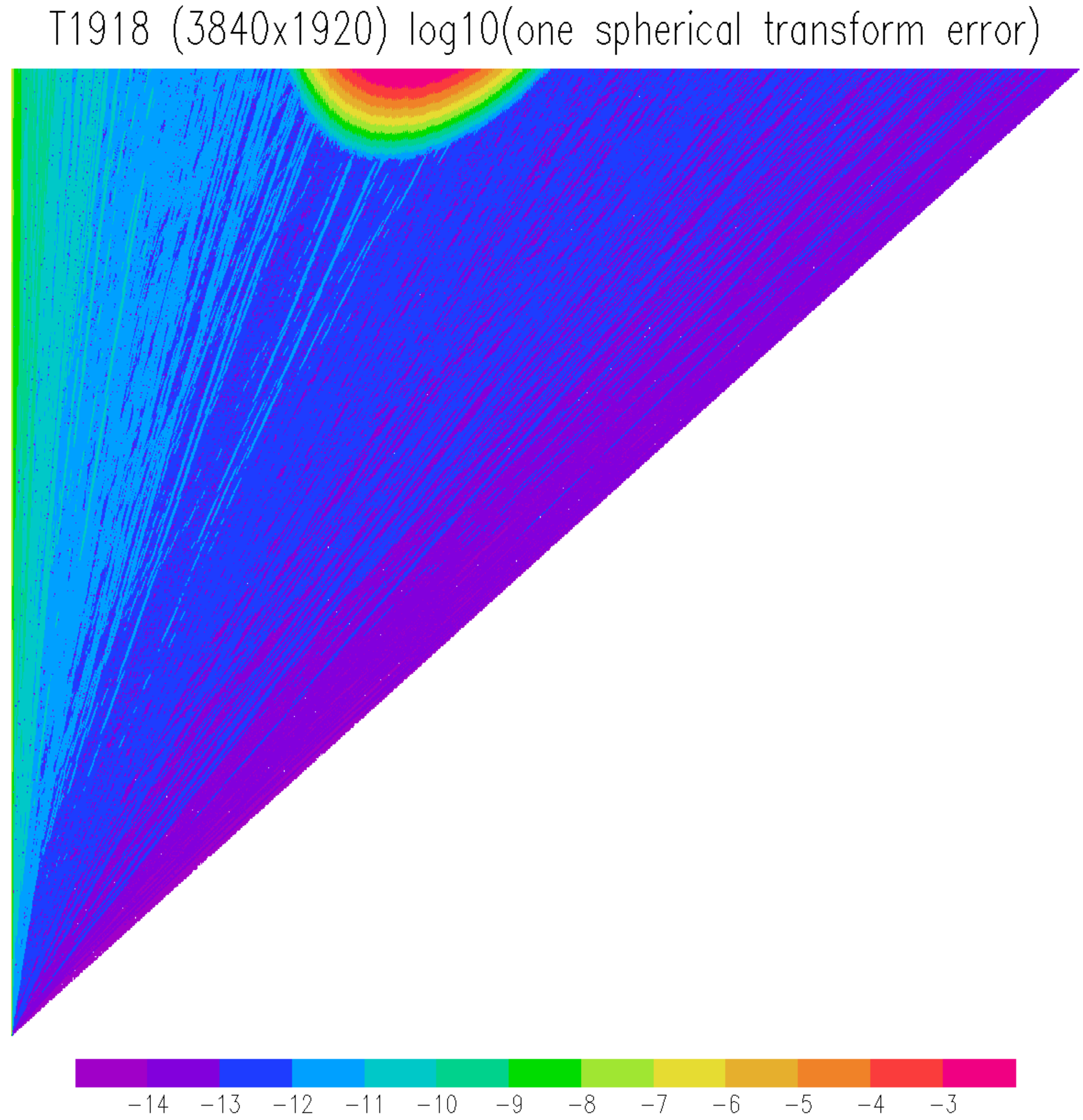
Absolute difference between preset and one complete spherical transform

Based on sp lib

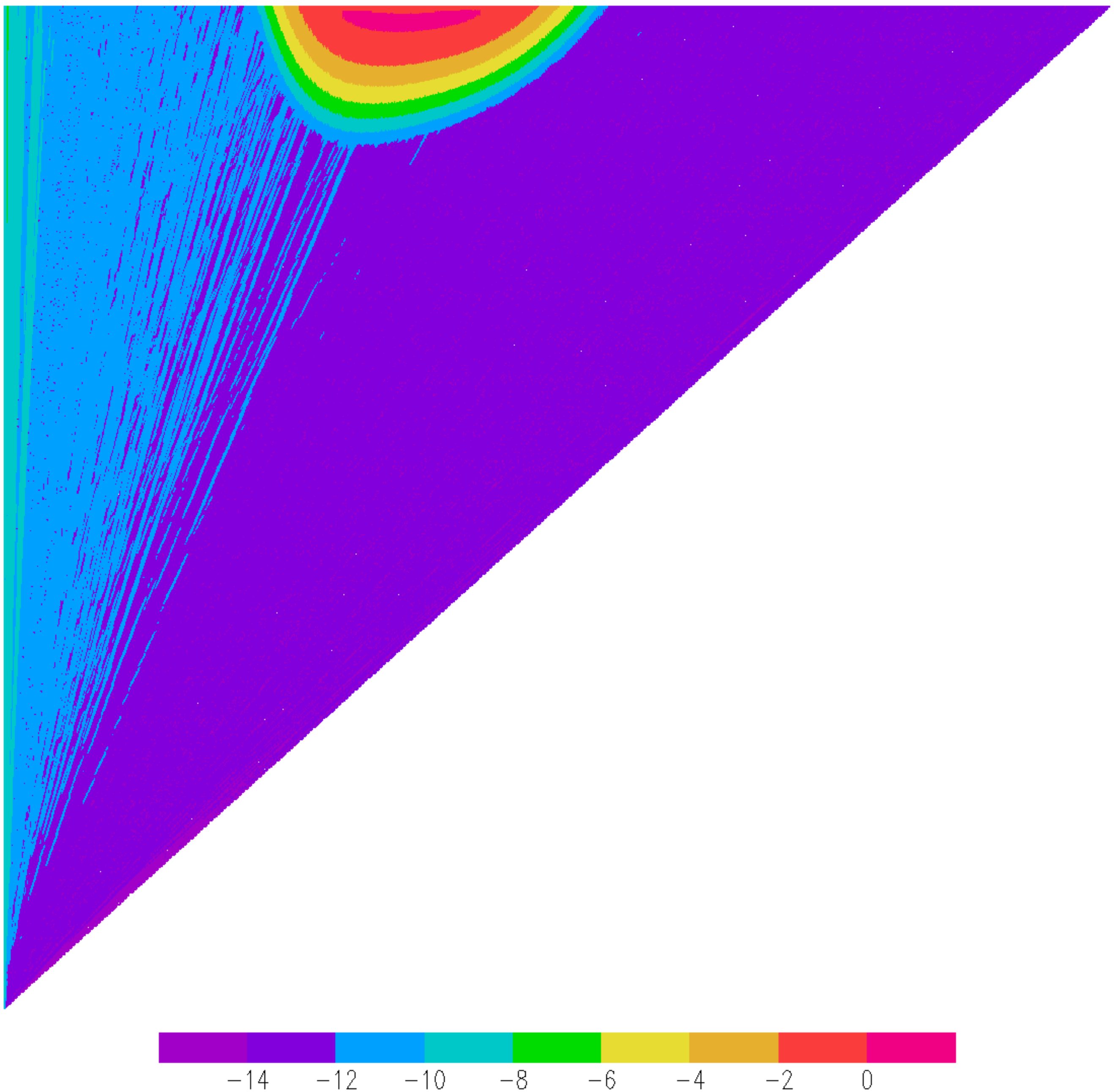
spectral	grid	Abs max error	location	note
878	2640x1320	3×10^{-8}	m=0,n=560	✔
1148	2304x1152	2.5×10^{-8}	m=0,n=789	✔
1756	3520x1760	4.9×10^{-8}	m=0,n=778	✔
1918	3840x1920	2.4×10^{-2}	m=714,n=1918	✗
2000	4032x2016	1.3	m=704,n=1968	✗
3000	6048x3024	1.3	m=333,n=2917	✗



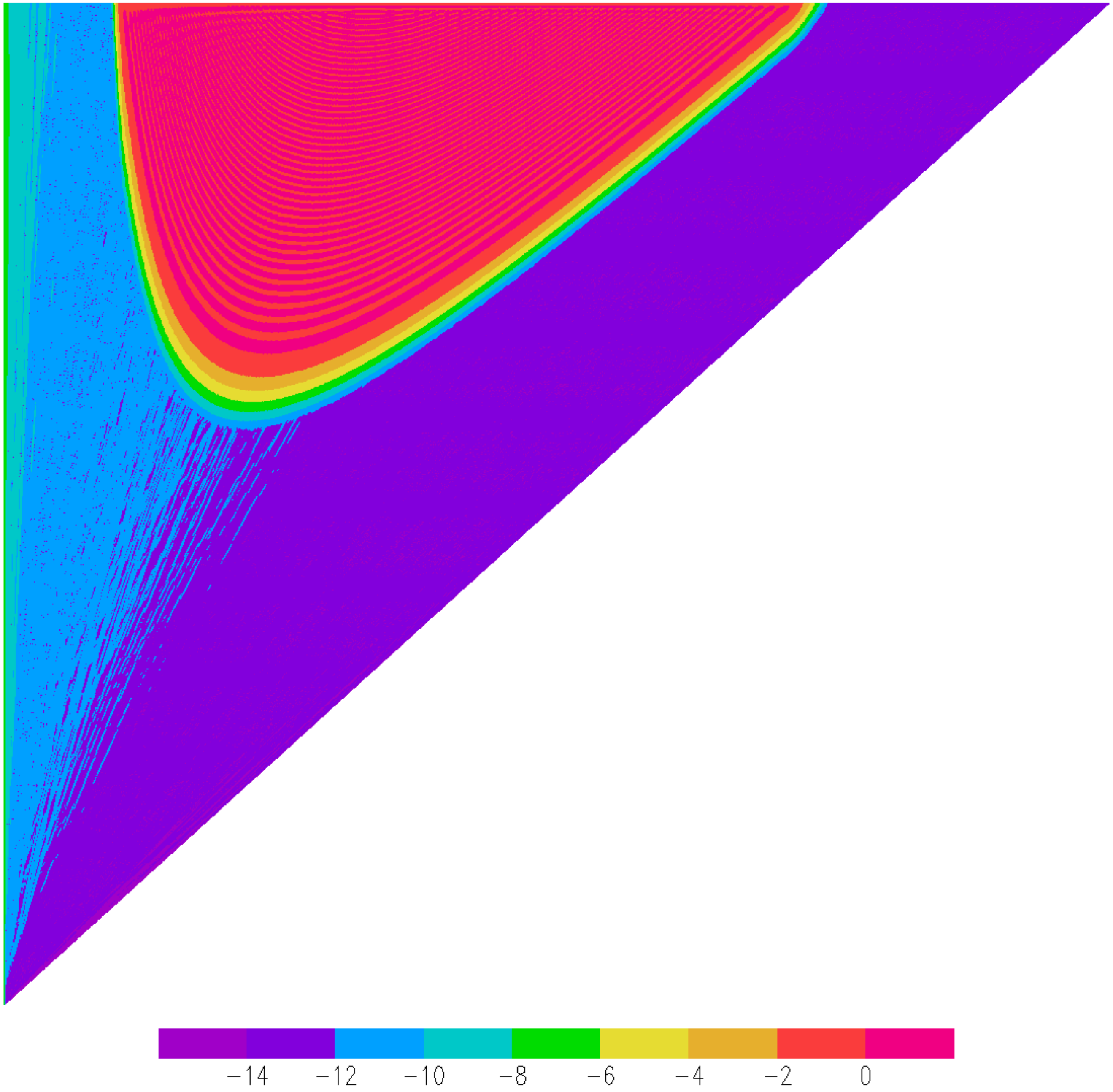
T1148 (2304x1152) log10(one spherical transform error)
−15
−14
−13
−12
−11
−10
−9
−8


T1918 (3840x1920) log10(one spherical transform error)
−14
−13
−12
−11
−10
−9
−8
−7
−6
−5
−4
−3

T2000 (4032x2016) log10(one spherical transform error)



T3000 (6144x3072) log10(one spherical transform error)



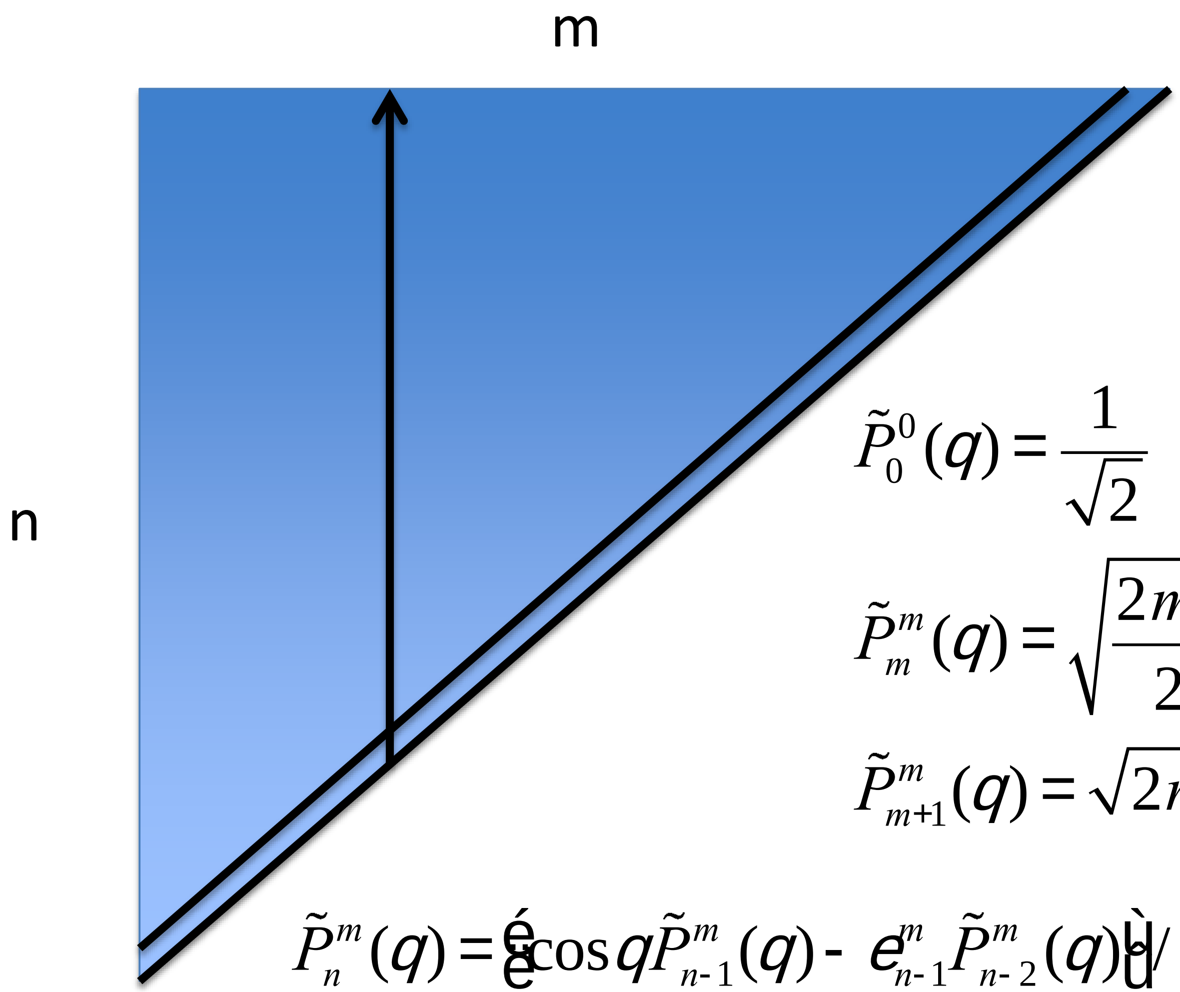
The reason

- Check with Spherical spectral transform

$$F^m(\mu_j) = \sum_{n=m}^{N} W_n^m(\mu_j) P_n^m(\mu_j)$$

$$W_n^m = \sum_{j=1}^{J} F^m(\mu_j) P_n^m(\mu_j) w_j$$

- Problem from the accurate of the Legender Polynomial function P(n,m)
- The P(n,m) are sequentially computed
- Errors come from initial P and long sequential computation



$$\tilde{P}_0^0(\theta) = \frac{1}{\sqrt{2}}$$

$$\tilde{P}_m^m(\theta) = \sqrt{\frac{2m+1}{2m}}\sin\theta \tilde{P}_{m-1}^{m-1}(\theta)$$

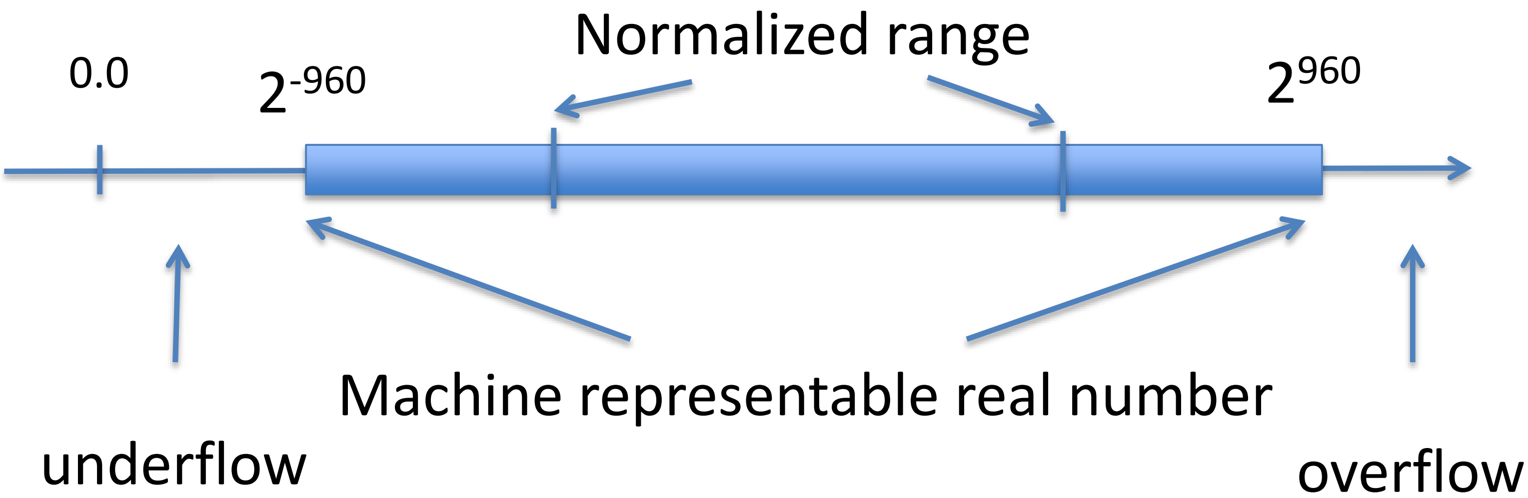
$$\tilde{P}_{m+1}^m(\theta) = \sqrt{2m+3}\cos\theta \tilde{P}_m^m(\theta)$$

$$\tilde{P}_n^m(\theta) = \left[\cos\theta \tilde{P}_{n-1}^m(\theta) - \varepsilon_{n-1}^m \tilde{P}_{n-2}^m(\theta)\right] / \varepsilon_n^m$$

$$\varepsilon_n^m = \sqrt{\frac{n^2 - m^2}{4n^2 - 1}}$$

Introduce solution

- Computation in higher accurate
- We have already used double precision
- quadruple precision etc cost much and always have limitation
- ECMWF and others have their own ways to fix
- Introduce X-number
  - Any number f can be represented as f=xB**I
  - B is 2**960 for real*8



X number takes care of under and over flow by xB**I with B=2**960, so xB**0 is machine representable real number but xB**(-1) represents underflow And xB**(+1) represents overflow. The details in Fukushima 2011

xB**ix time yB**iy, should be equal to x*yB**(ix+iy), but we should take care underflow by x*y, to do so, consider normalize of any X-number into the range of 2**(-480) and 2**(480). If x and y are normalized, then x*y will not be over- or under-flow.

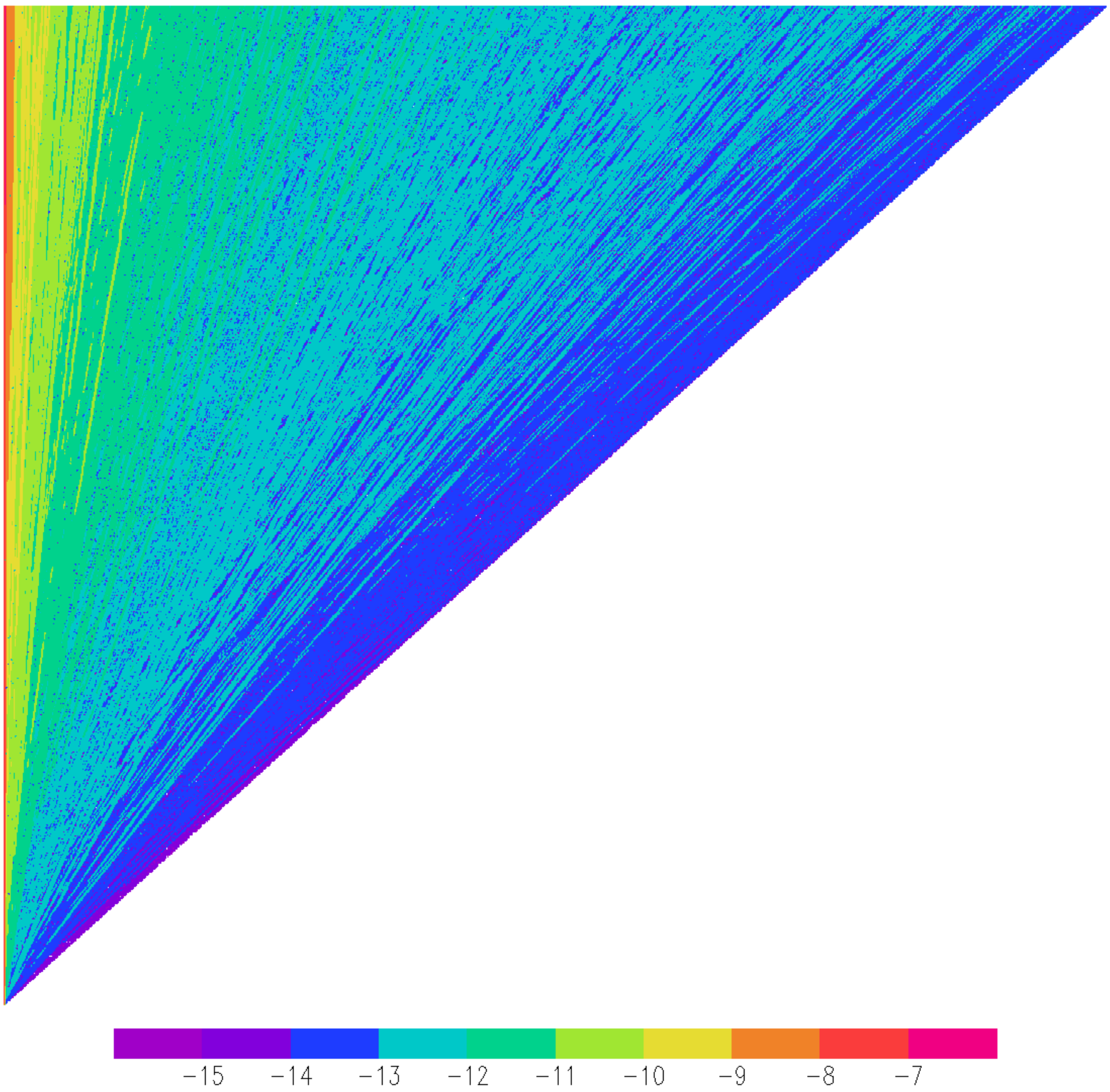
Implement X-number into SP lib, only in routine splegender

Absolute difference between preset and one complete spectral transform

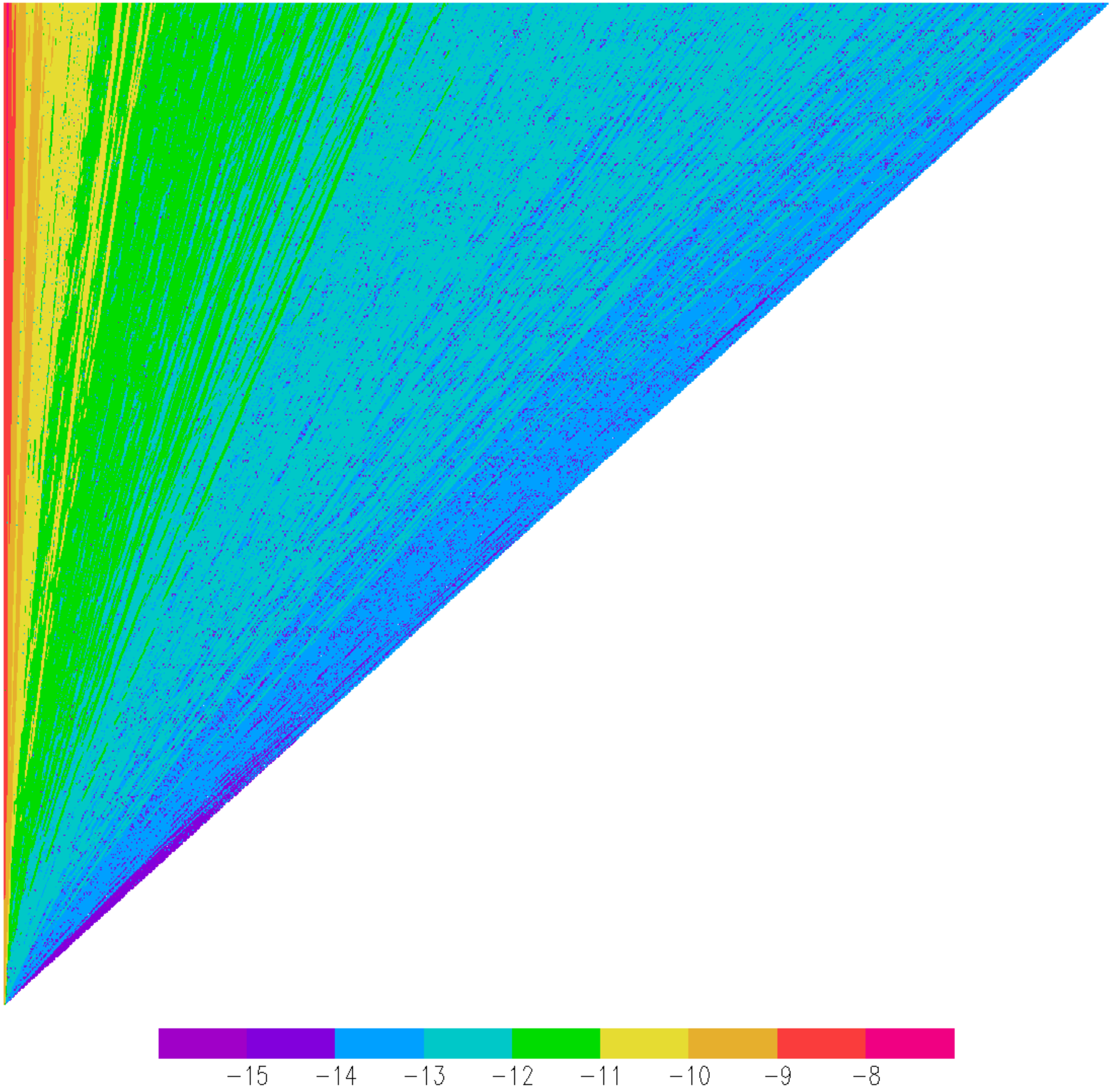
Based on sp lib + X-number for Legender

spectral	grid	Abs max error	location	note
878	2640x1320	$3x10^{-8}$	m=0,n=560	✔
1000	2016x1008	$1.5x10^{-9}$	m=0,n=789	✔
1148	2304x1152	$2.5x10^{-8}$	m=0,n=812	✔
2000	4032x2016	$1.2x10^{-8}$	m=0,n=1968	✔
3000	6048x3024	$5.1x10^{-7}$	m=0,n=1118	✔

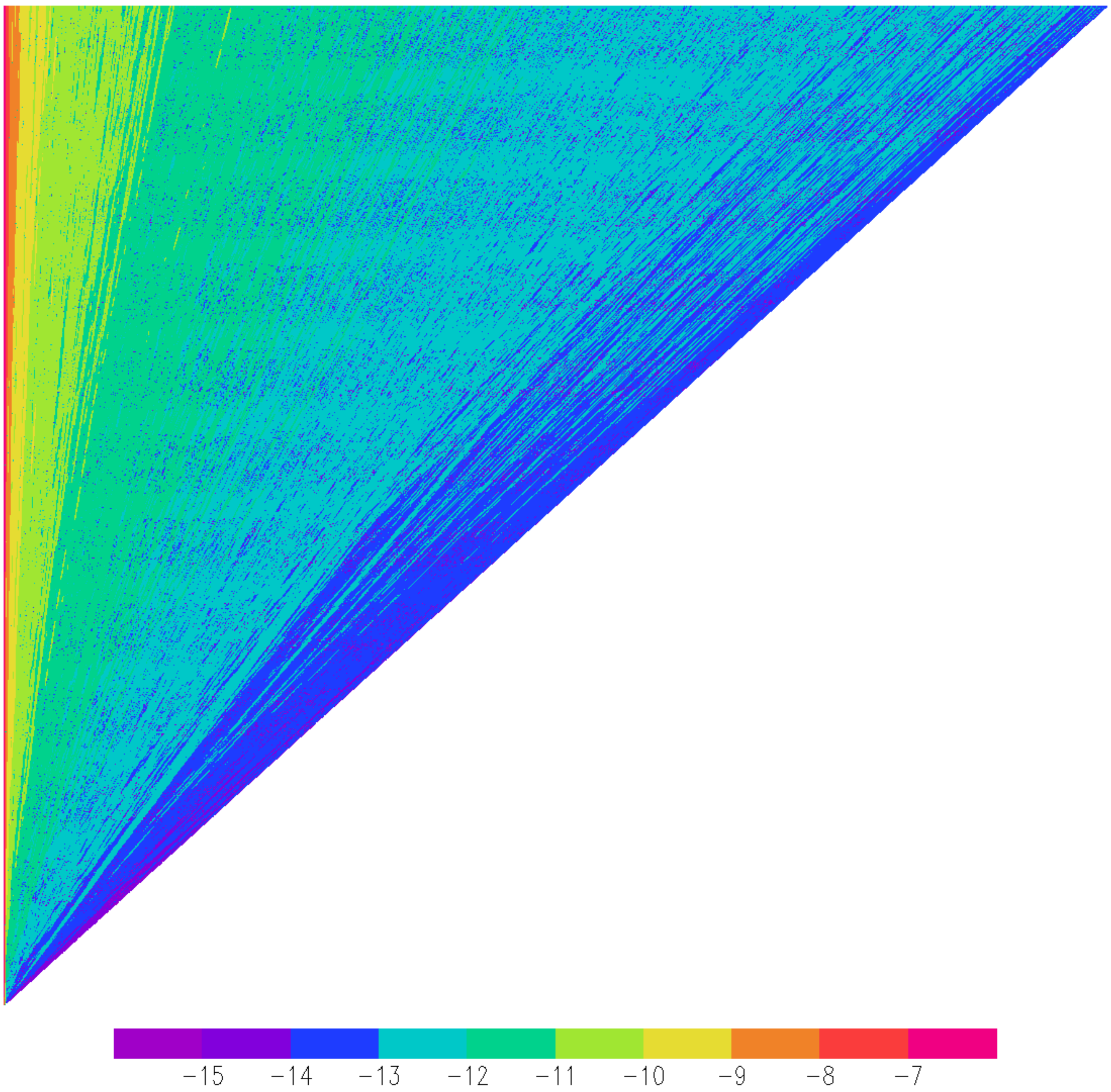
T1918 with X-number log10(one spherical transform error)



T2000 with X-number log10(one spherical tranform error)



T3000 with X-number log10(one spherical transform error)



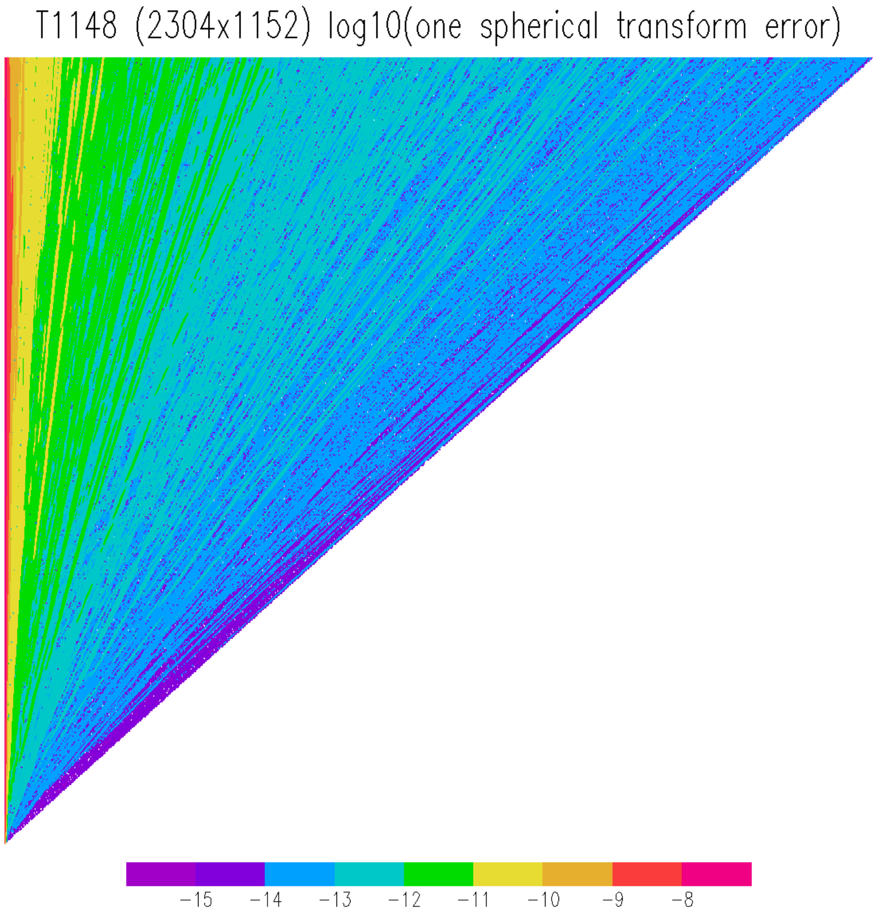
How about low m accuracy?

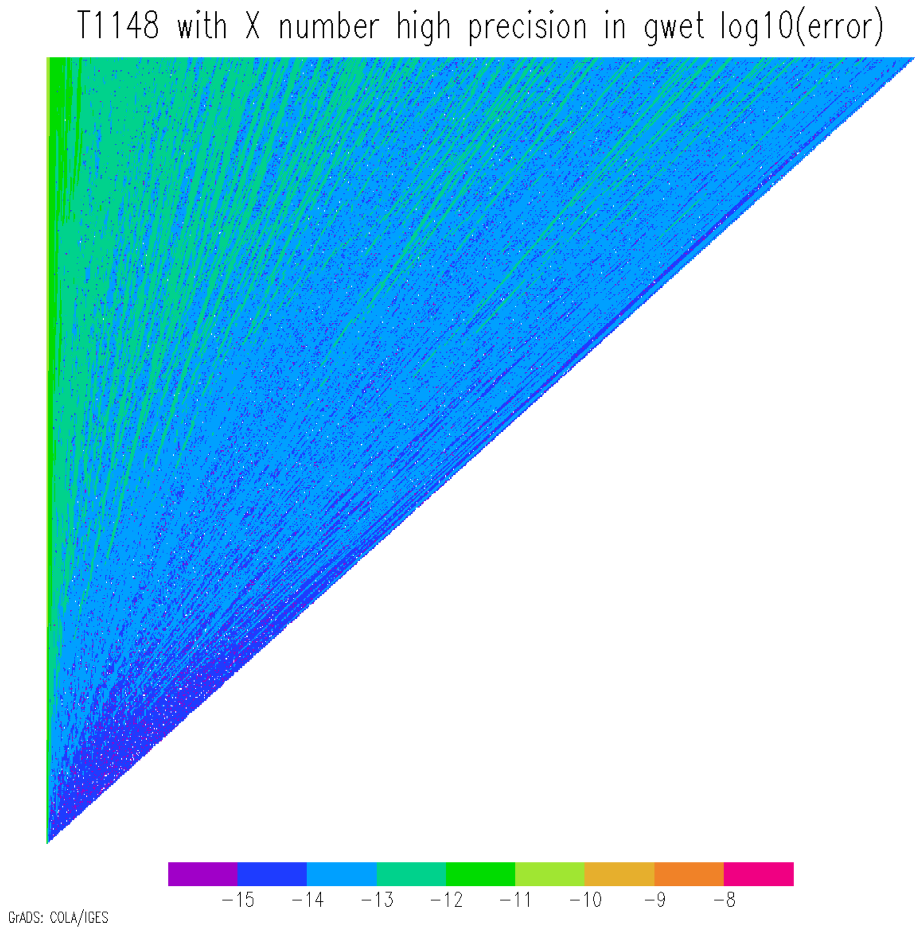
- Along m=0, 1, and/or 2 and all n have less accurate than others
- The reason is due to less accurate Gaussian weight in spherical transform
- So only the routine computing Gaussian weight we change real kind from (15,45) to (30,90)
- Note that (20,60) has the same results

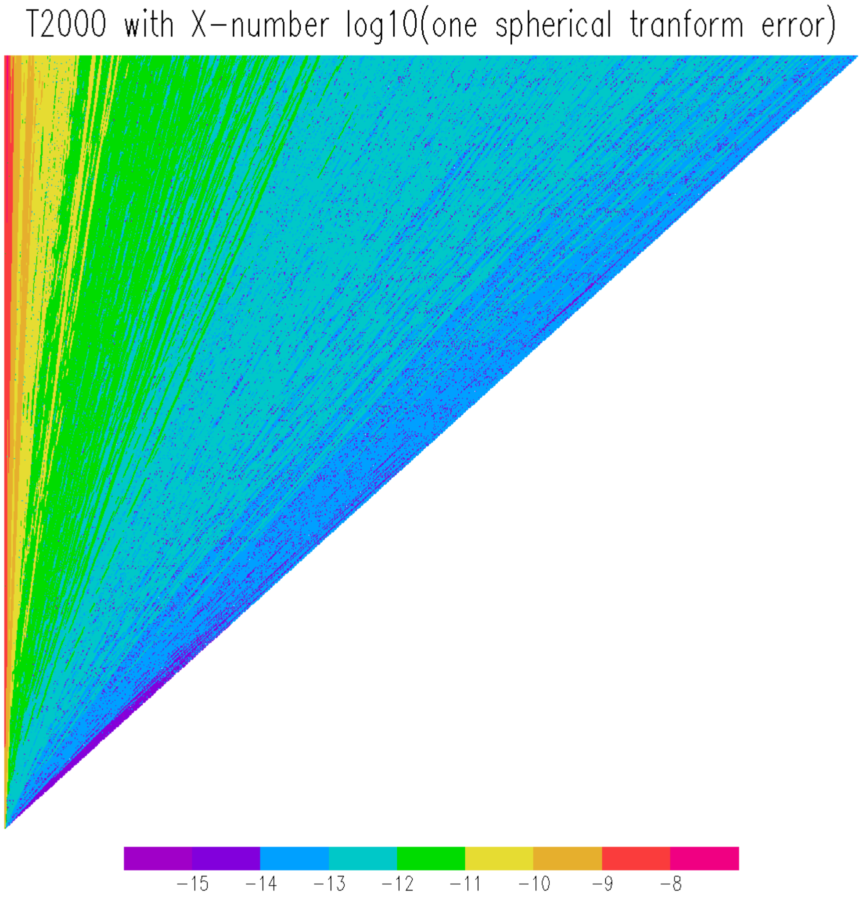
Absolute difference between preset and one complete spectral transform

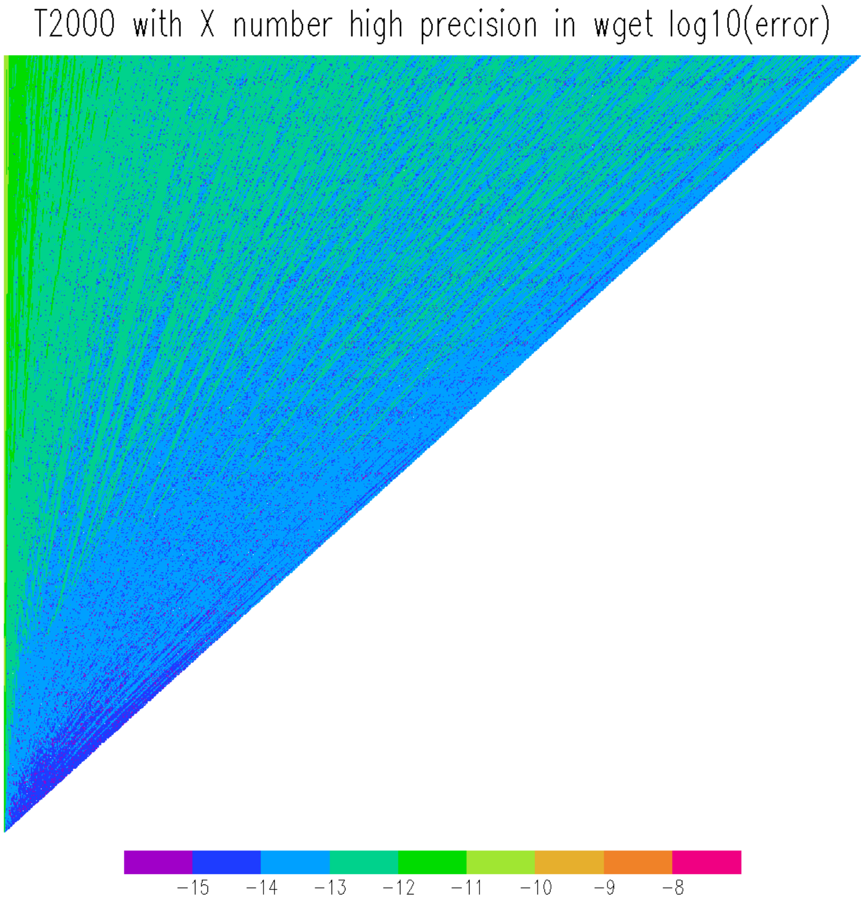
Compare X number and splat with (15,45) or (30,90)

spectral	grid	|max error| X+(15,45)	|max error| X+(30,90)	note
878	2640x1320	3.0×10^{-8}	2.3×10^{-11}	✔
1000	2016x1008	1.5×10^{-9}	8.2×10^{-12}	✔
1148	2304x1152	2.5×10^{-8}	2.6×10^{-11}	✔
2000	4032x2016	1.2×10^{-8}	3.0×10^{-11}	✔
3000	6048x3024	5.1×10^{-7}	3.6×10^{-11}	✔


T1148 (2304x1152) log10(one spherical transform error)
−15
−14
−13
−12
−11
−10
−9
−8

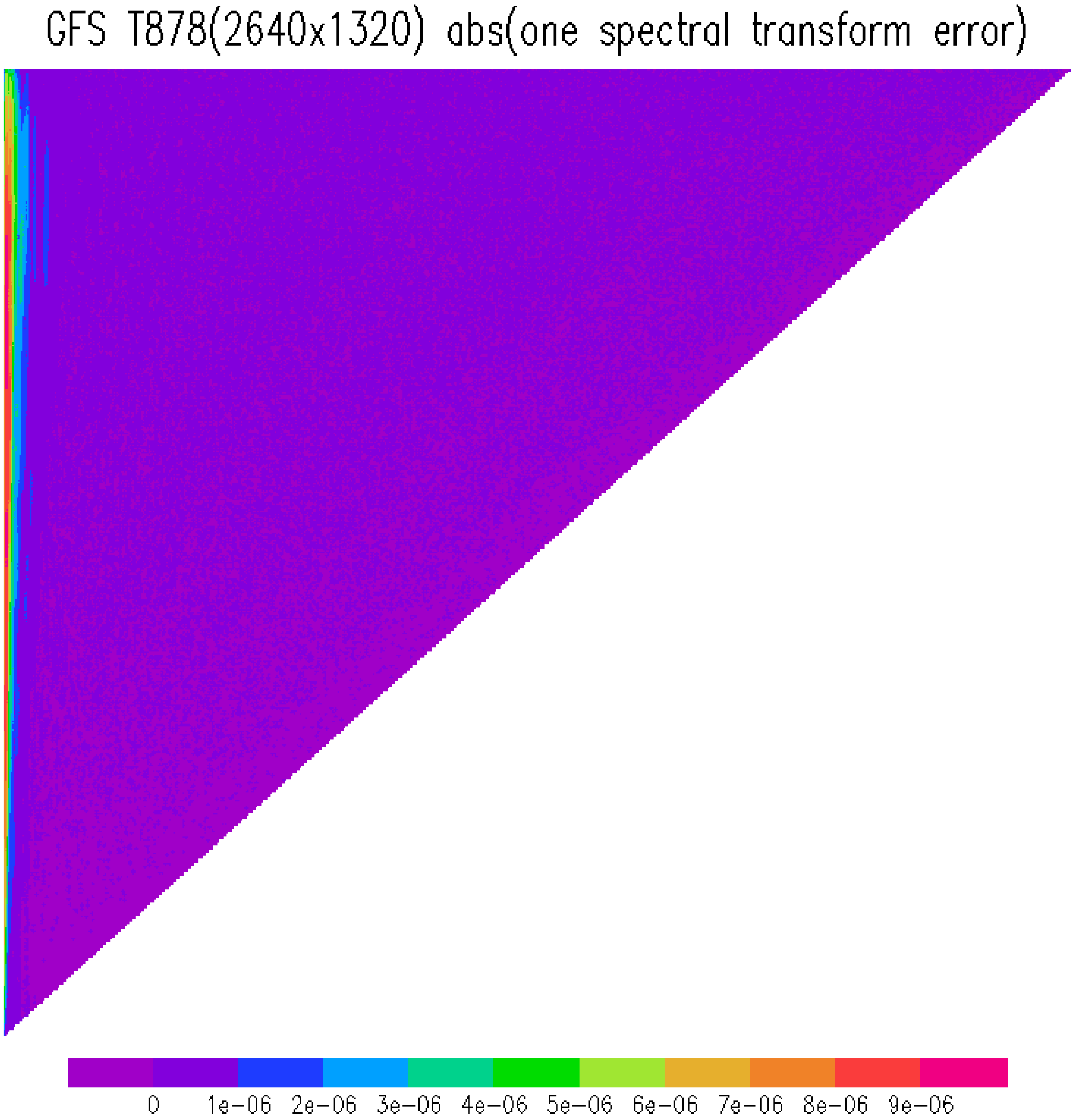

T1148 with X number high precision in gwet log10(error)
−15
−14
−13
−12
−11
−10
−9
−8
GrADS: COLA/IGES


T2000 with X-number log10(one spherical tranform error)
-15 -14 -13 -12 -11 -10 -9 -8

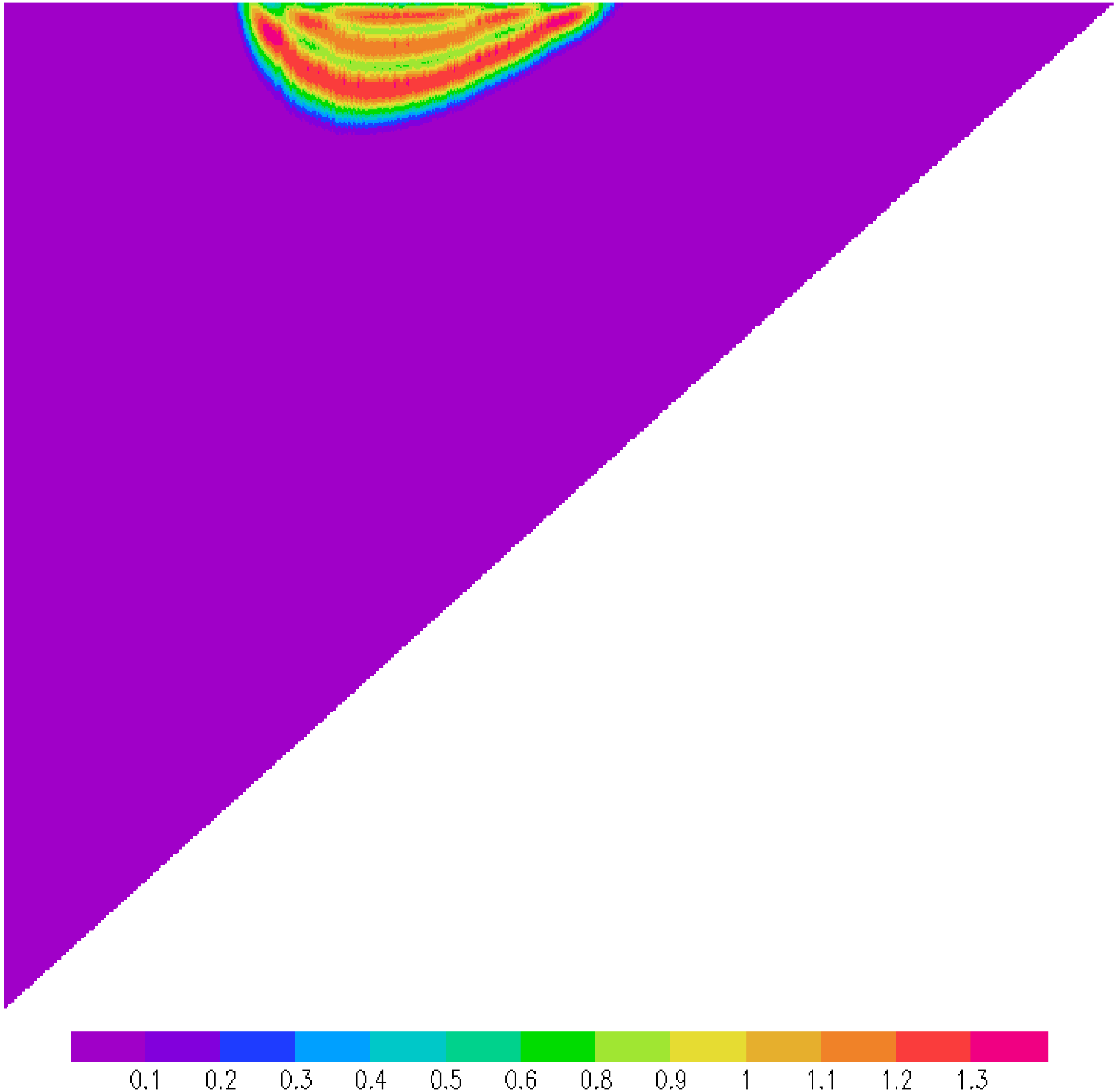

T2000 with X number high precision in wget log10(error)
-15 -14 -13 -12 -11 -10 -9 -8

Examine inside the GSM

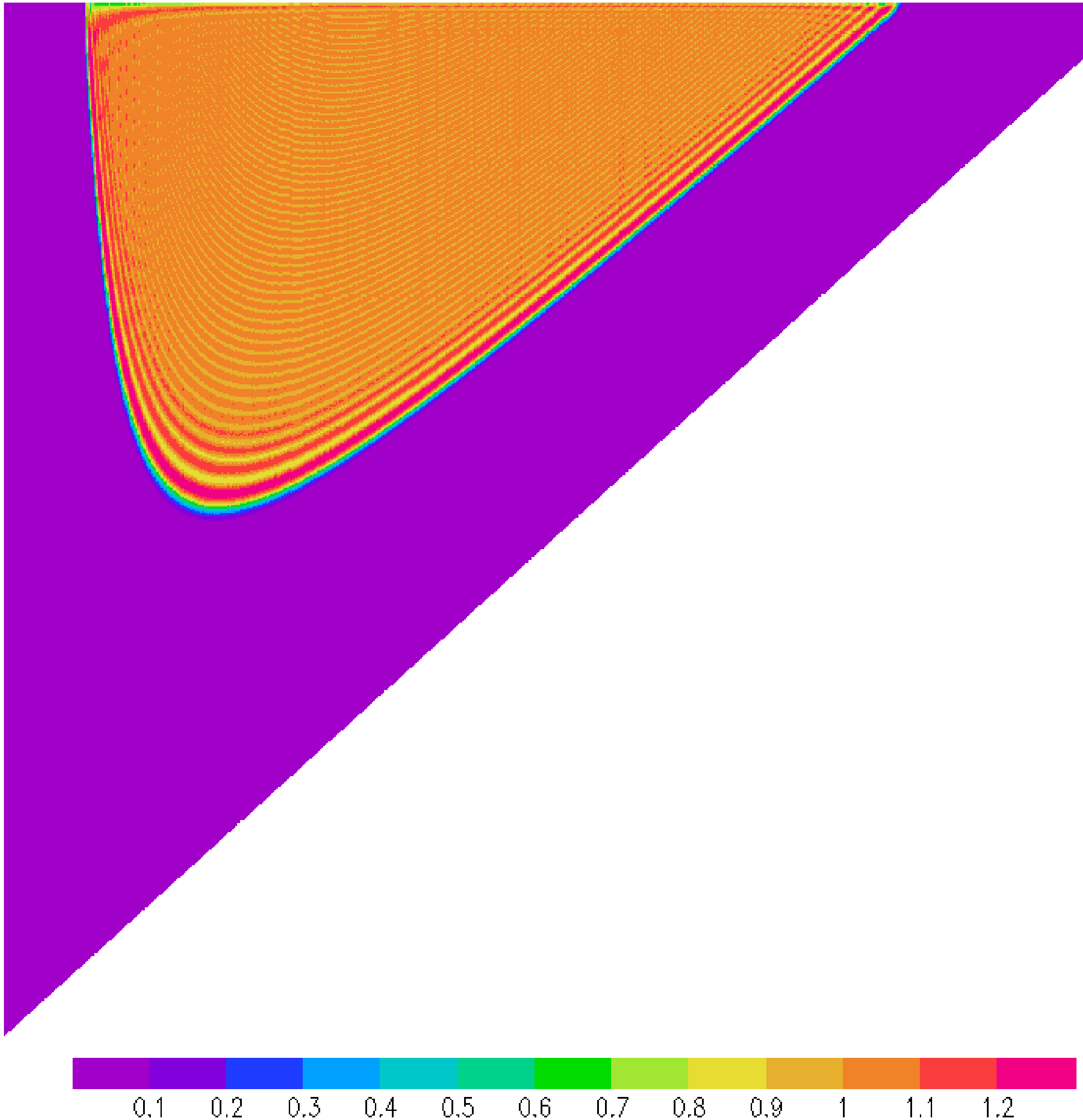
- Use X-number to take care underflow for Legendre polynomial function in sp lib fixes the problem of spherical harmonic transform by using T2000 & above
- Increase precision only in computing Gaussian weight helps accuracy by order of 3
- Next step is to check GSM (NCEP GFS) and fix it if necessary

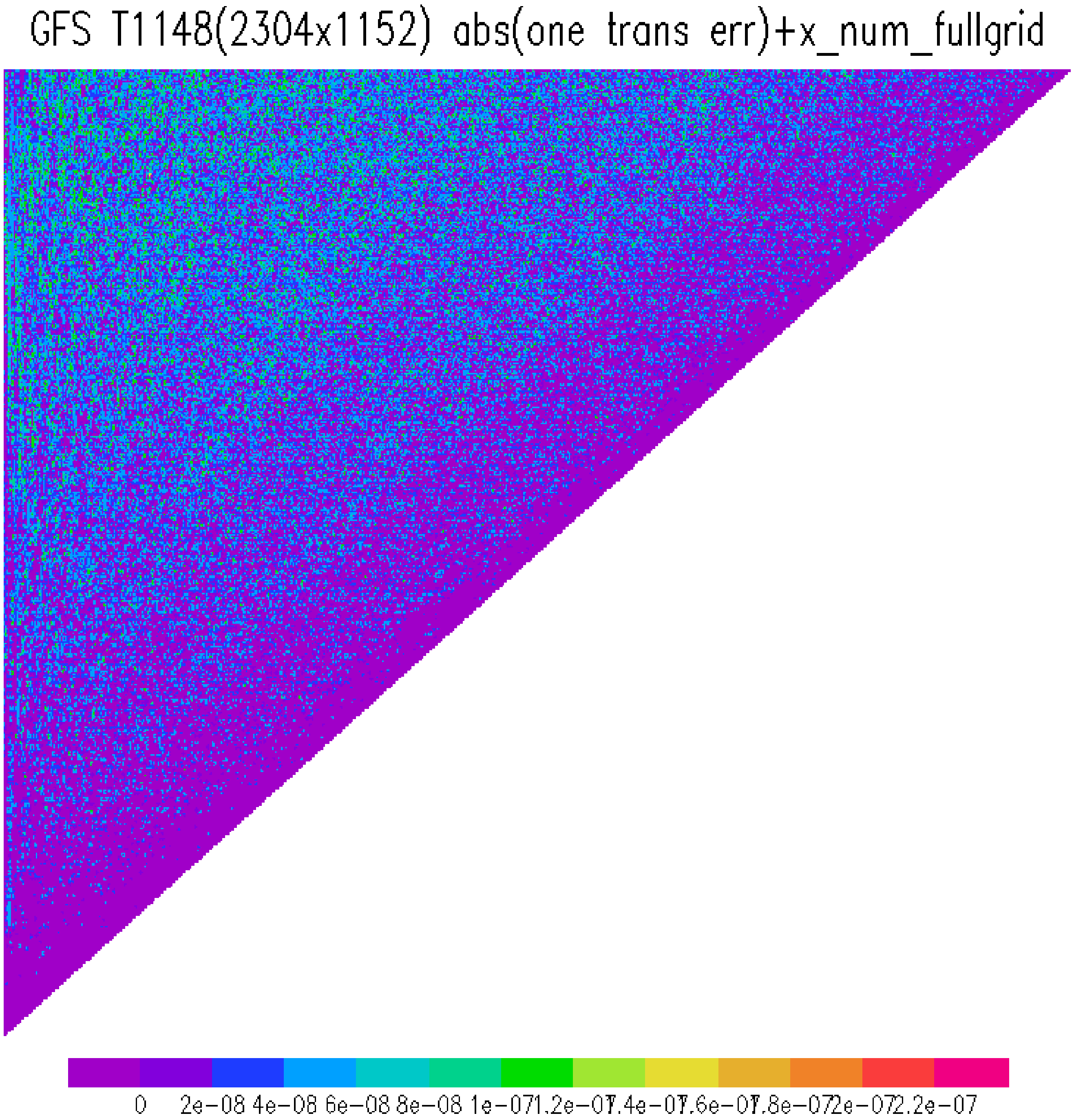

GFS T878(2640x1320) abs(one spectral transform error)
0
1e-06
2e-06
3e-06
4e-06
5e-06
6e-06
7e-06
8e-06
9e-06

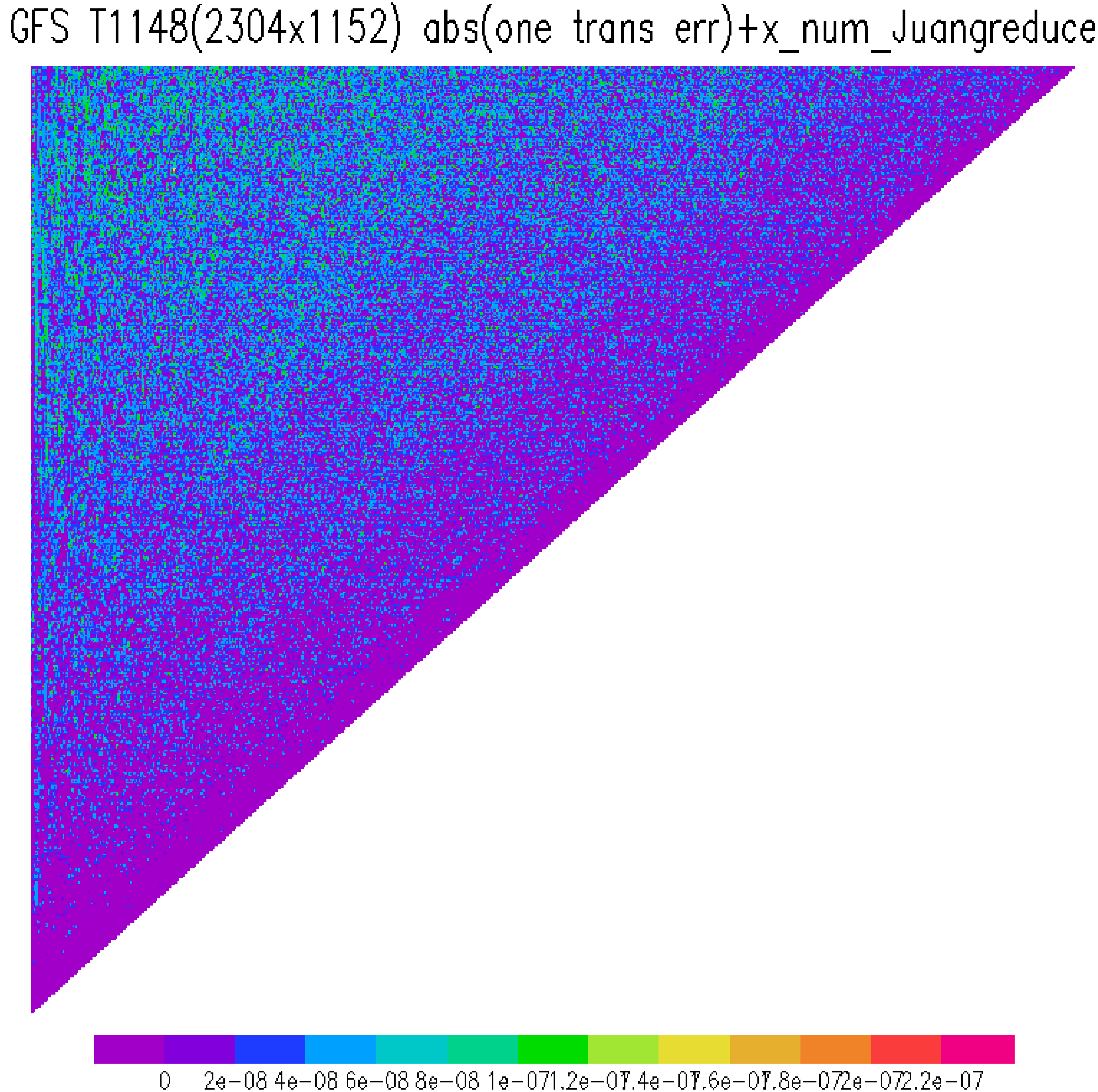
GFS T1148(2304x1152) abs(one spectral transform error)



GFS T2000(4032x2016) abs(one spectral transform error)



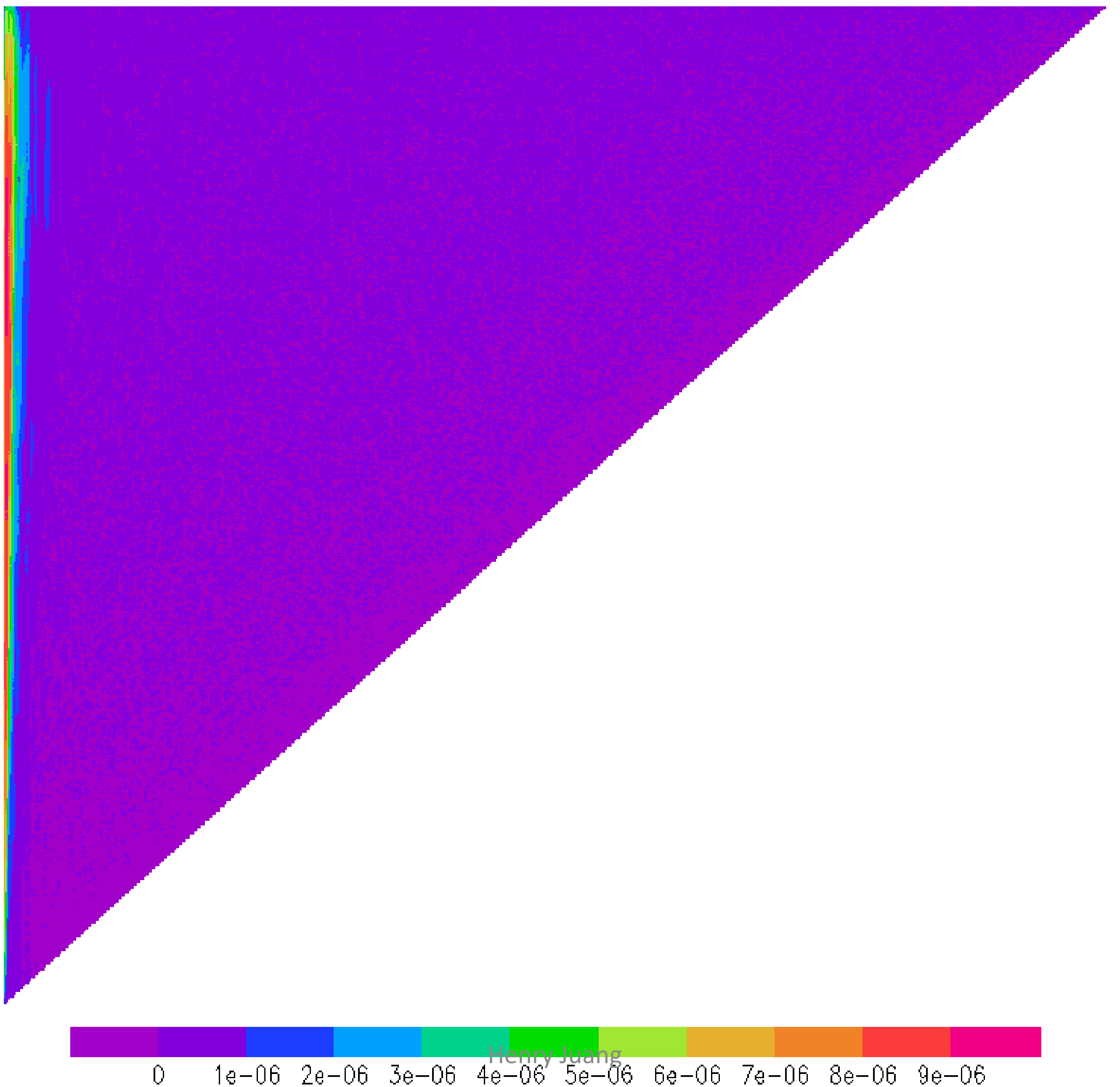

GFS T1148(2304x1152) abs(one trans err)+x_num_fullgrid
0
2e−08
4e−08
6e−08
8e−08
1e−07
1.2e−07
1.4e−07
1.6e−07
1.8e−07
2e−07
2.2e−07

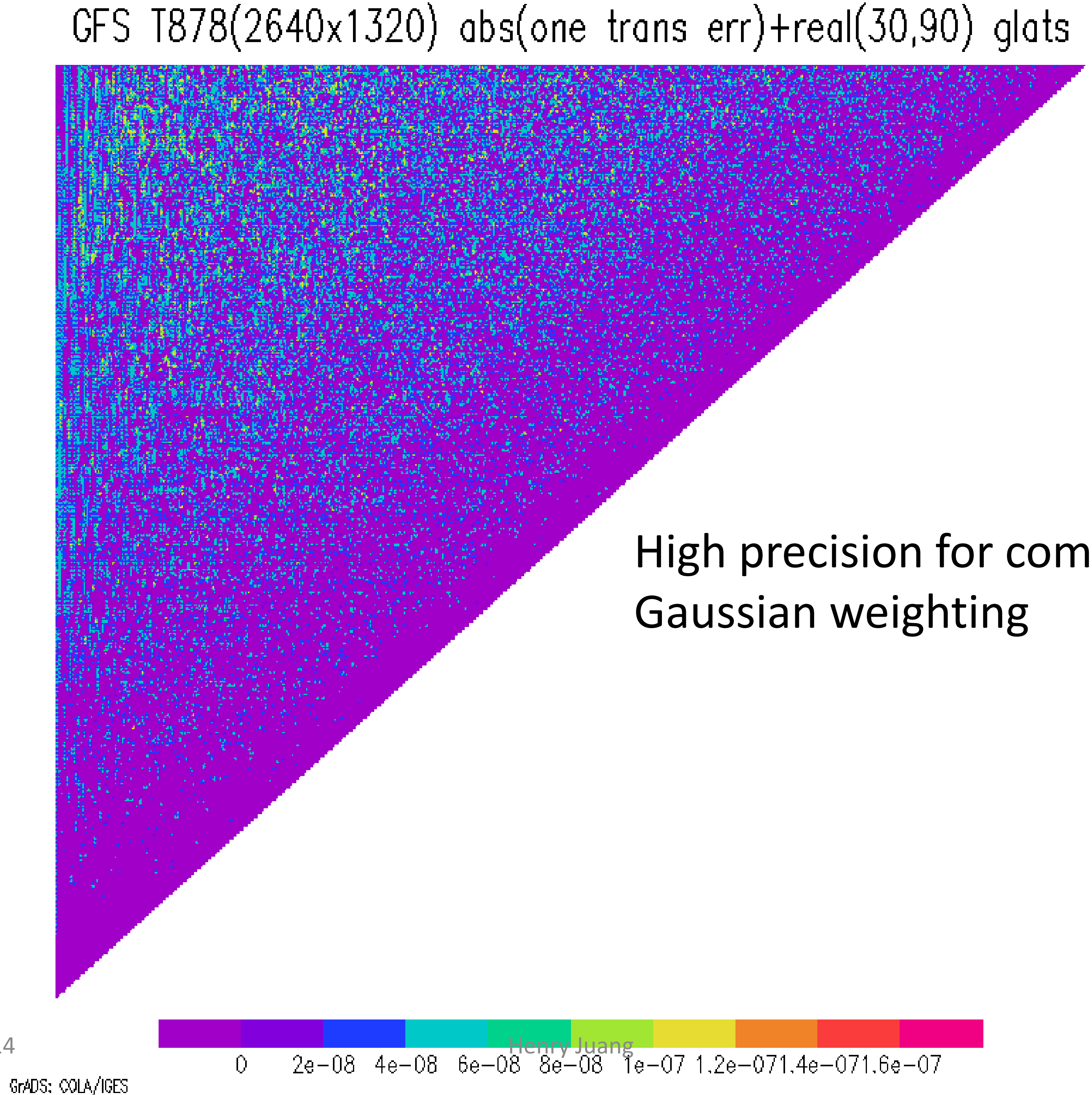

GFS T1148(2304x1152) abs(one trans err)+x_num_Juangreduce
0
2e-08
4e-08
6e-08
8e-08
1e-07
1.2e-07
1.4e-07
1.6e-07
1.8e-07
2e-07
2.2e-07

Apply to GSM

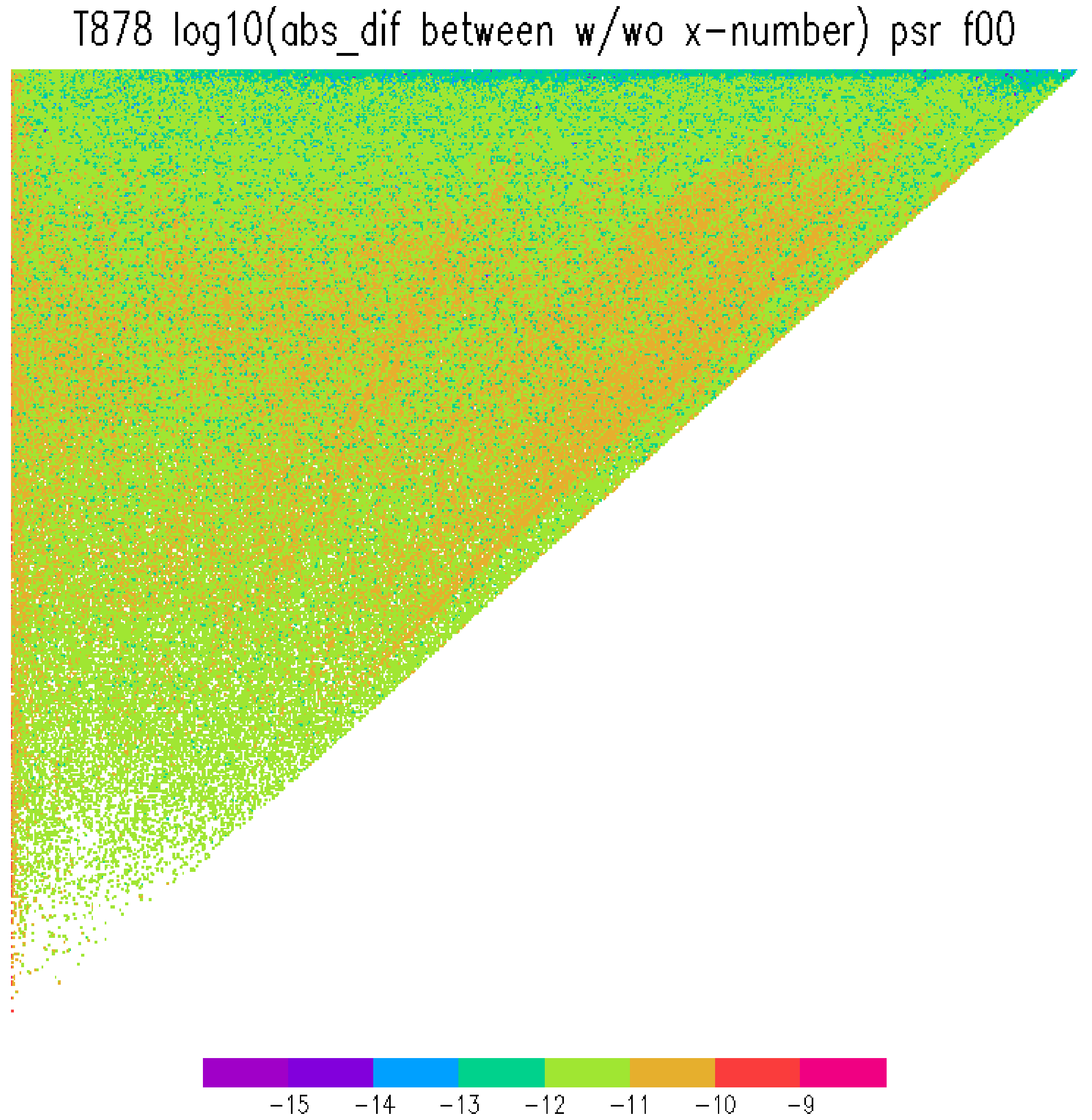
- We have applied higher precision (30,90) in Gaussian weighting and X-number to correct spherical harmonic transform (SHT) into opn version of GFS with test in one case with T878 of full Eulerian model.
- We examine output in spectral space so no extra error may be introduced by postprocessor.
- We assume the difference is the magnitude we can improve, since only higher precision and corrected Legendre function we provide.

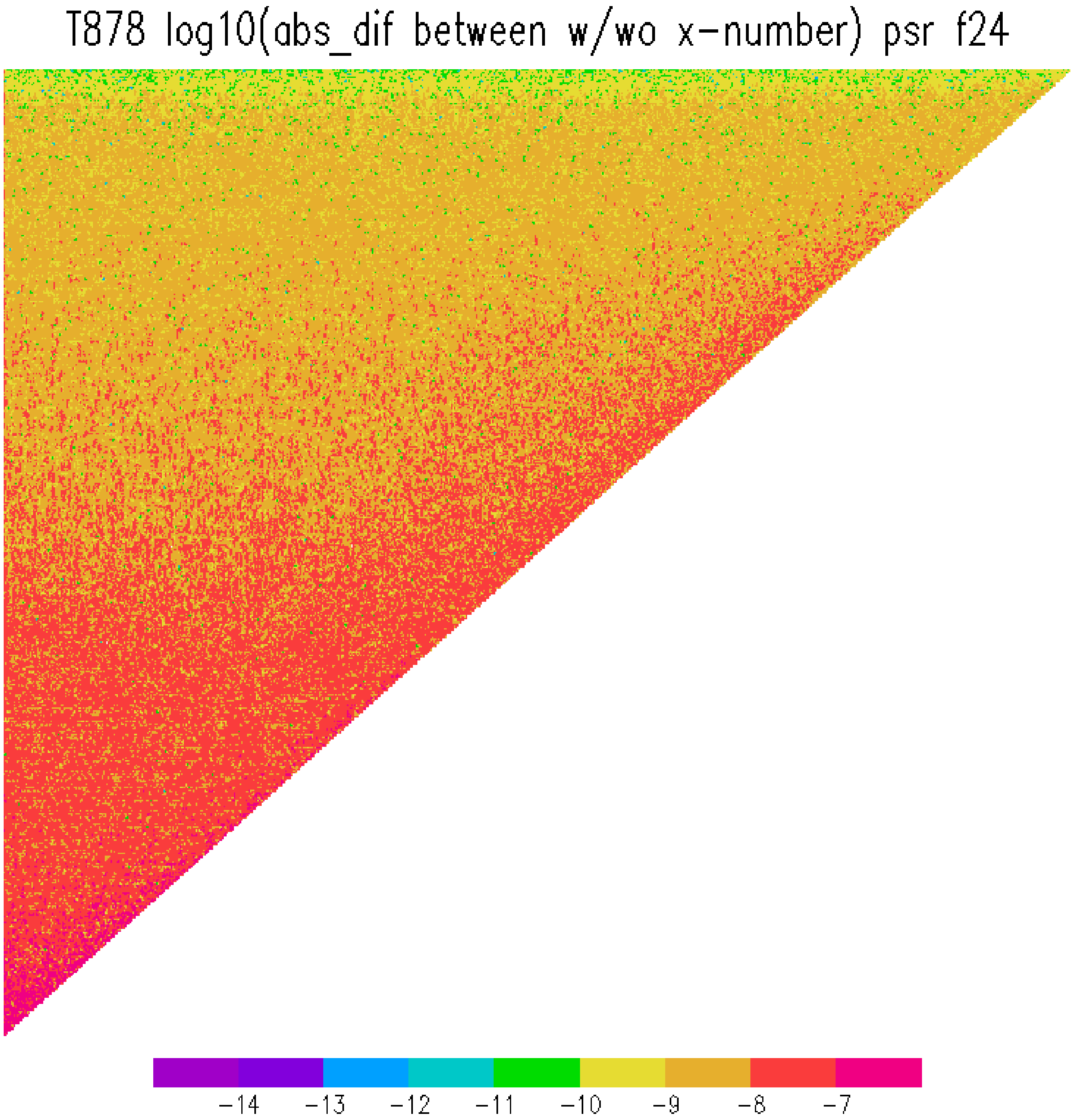
GFS T878(2640x1320) abs(one spectral transform error)

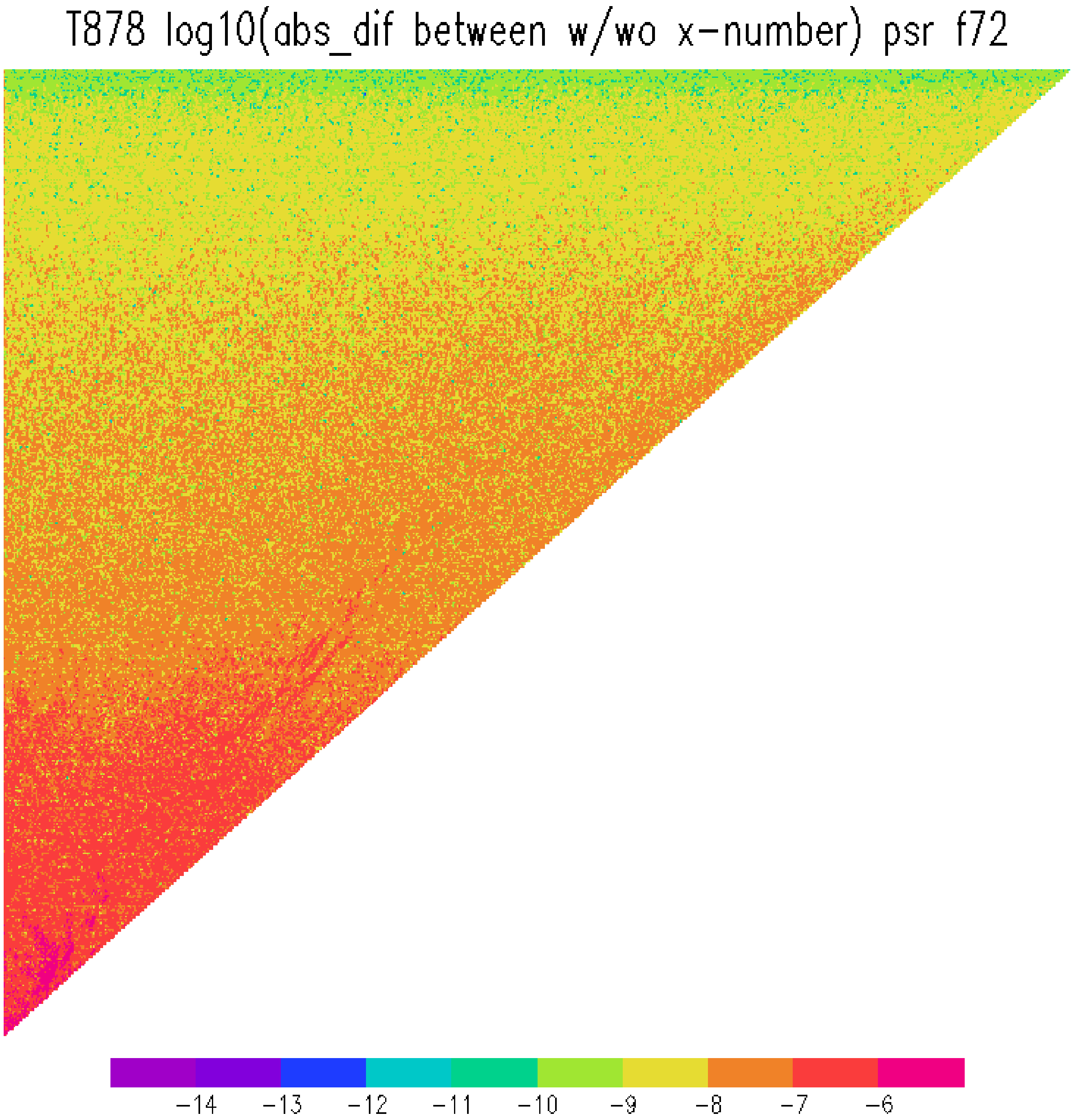


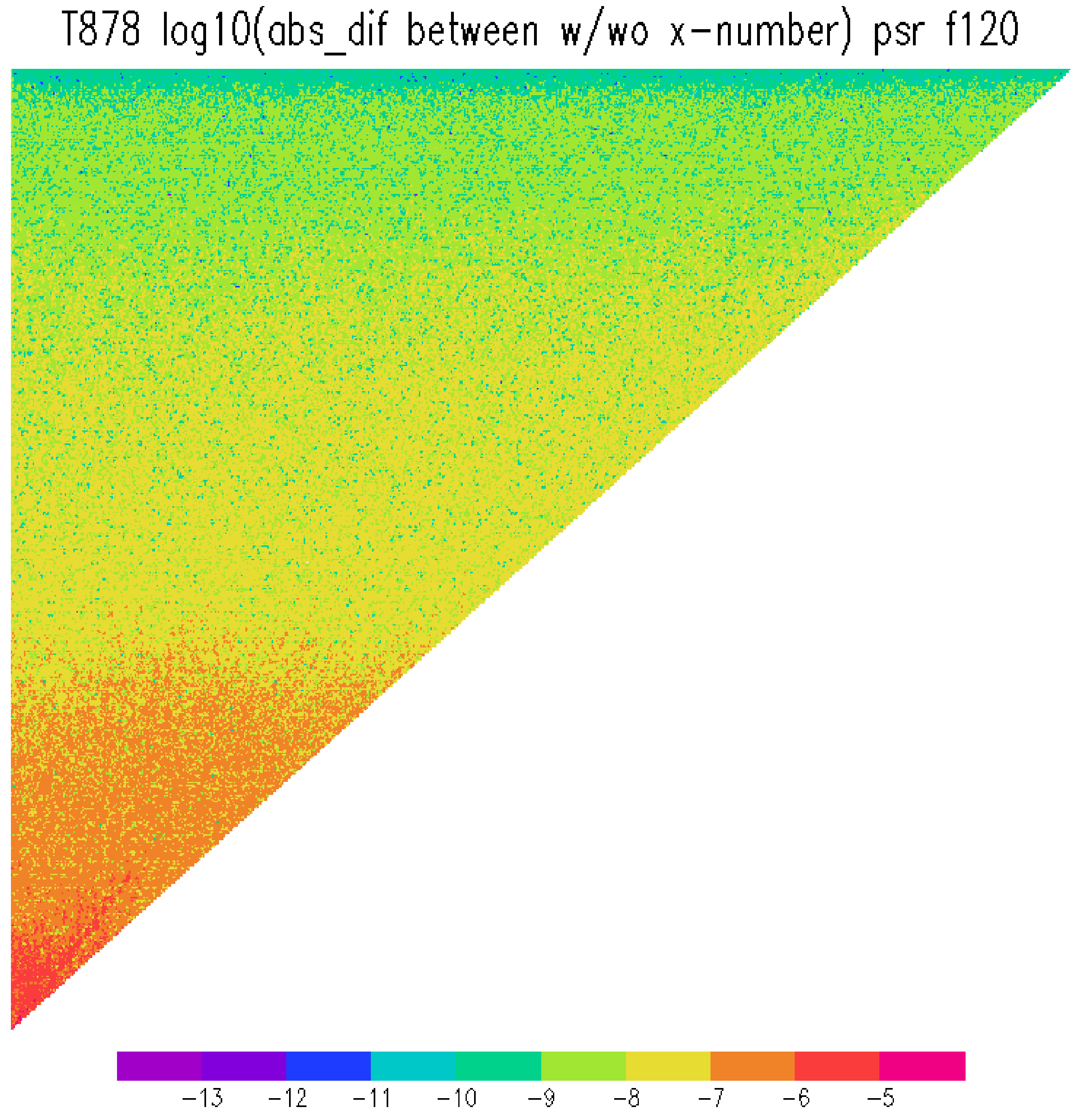


High precision for computing Gaussian weighting

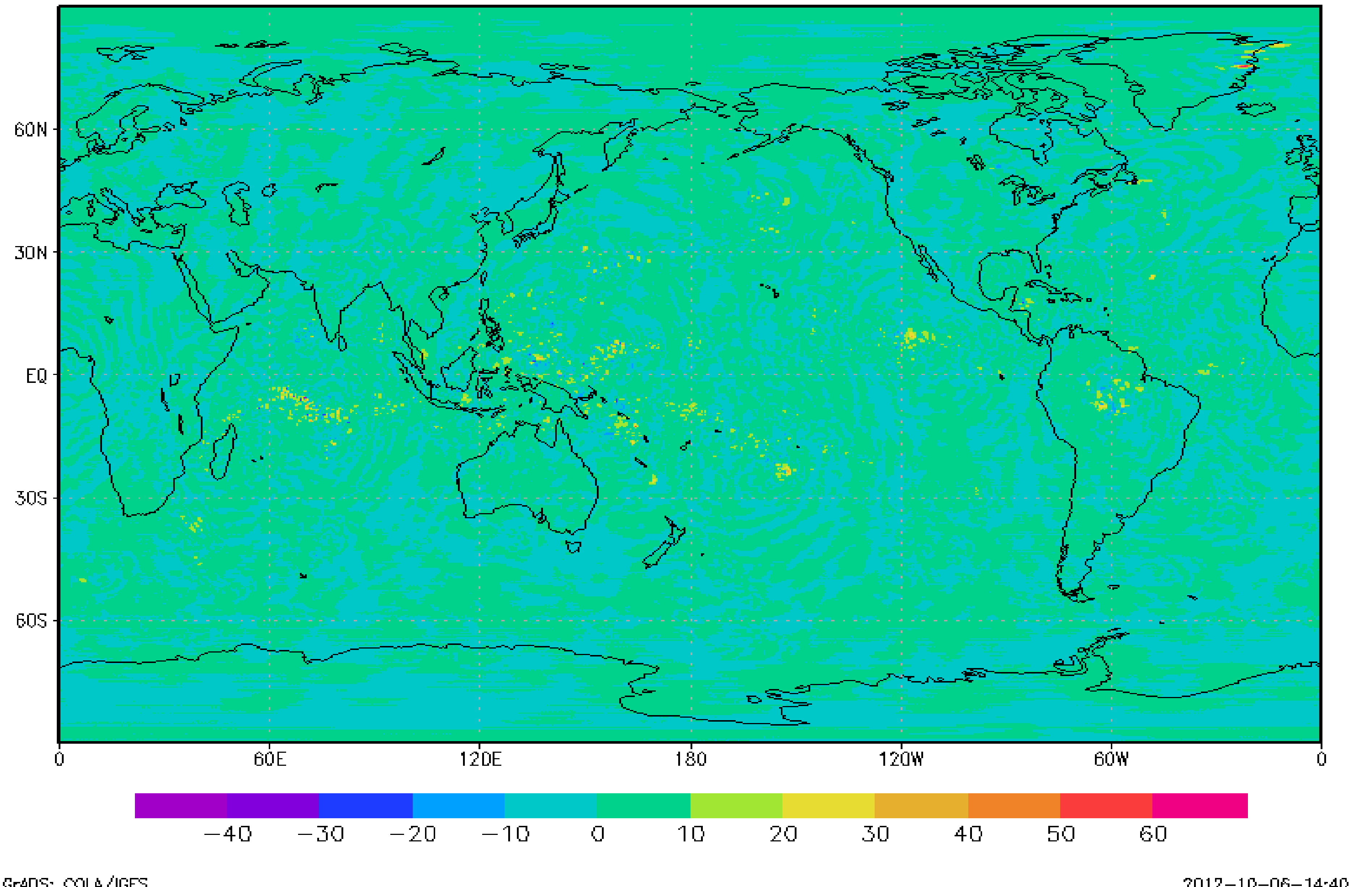

T878 log10(abs_dif between w/wo x-number) psr f00
−15
−14
−13
−12
−11
−10
−9

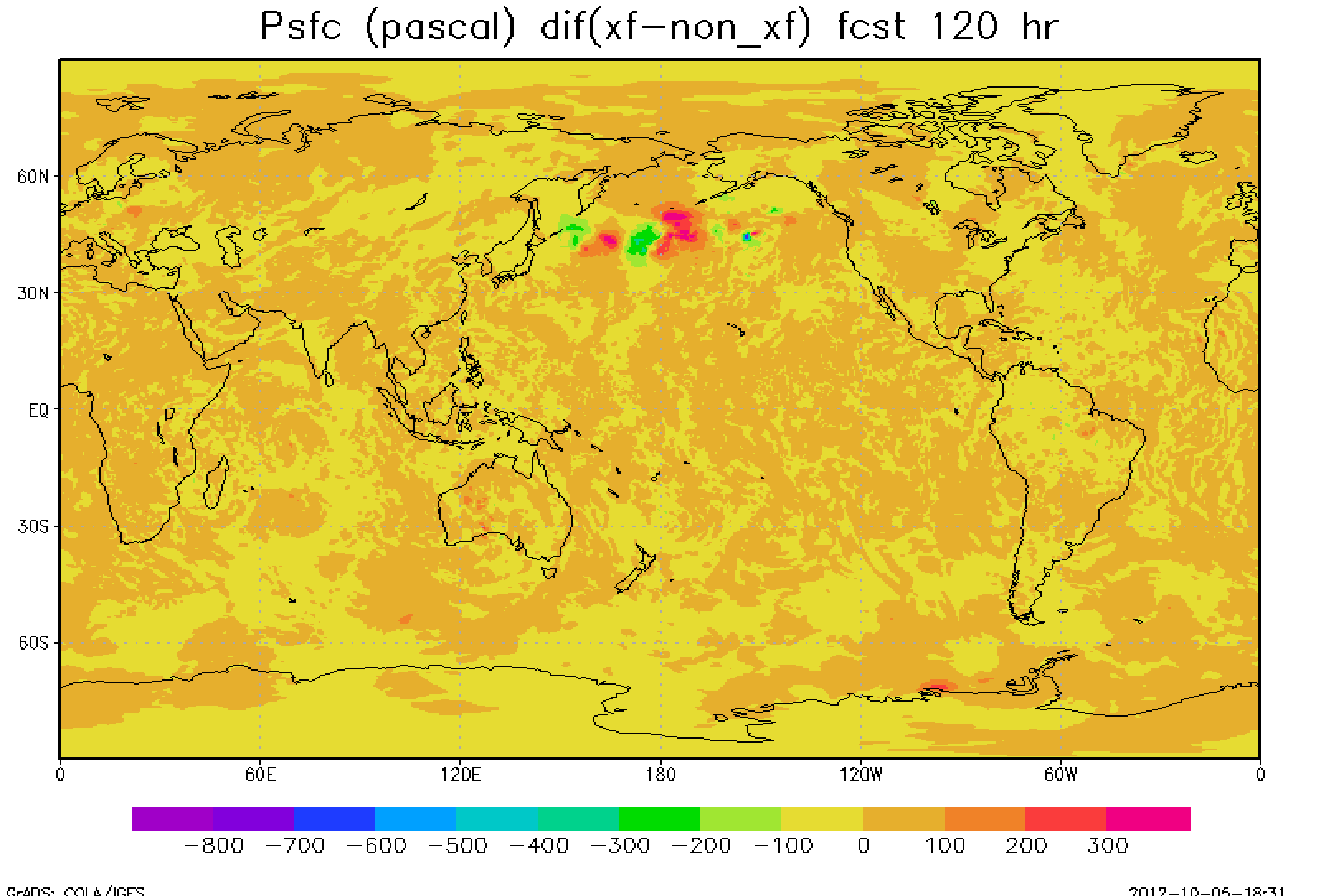

T878 log10(abs_dif between w/wo x-number) psr f24
-14
-13
-12
-11
-10
-9
-8
-7


T878 log10(abs_dif between w/wo x-number) psr f72
−14
−13
−12
−11
−10
−9
−8
−7
−6


T878 log10(abs_dif between w/wo x-number) psr f120
−13
−12
−11
−10
−9
−8
−7
−6
−5

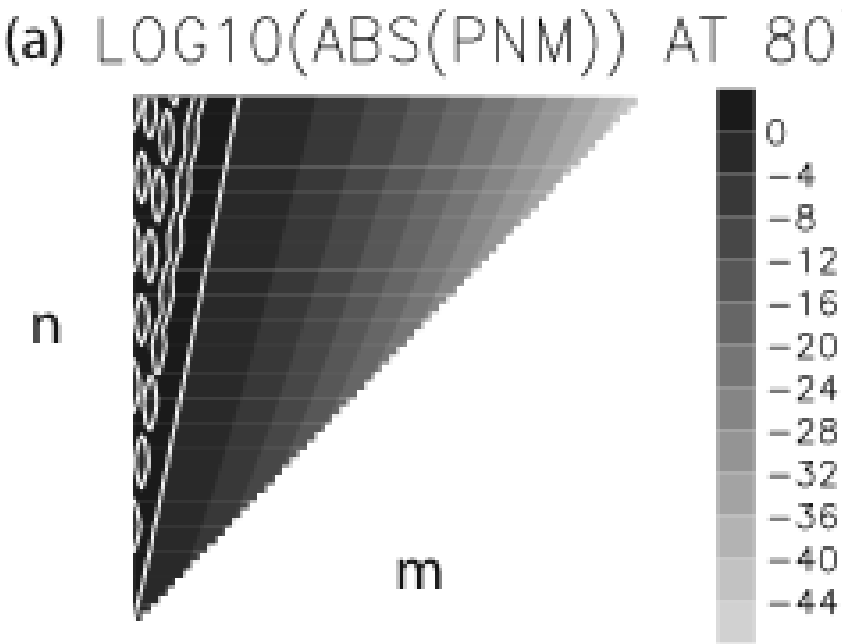
Psfc (pascal) dif(xf-non_xf) fcst 24 hr

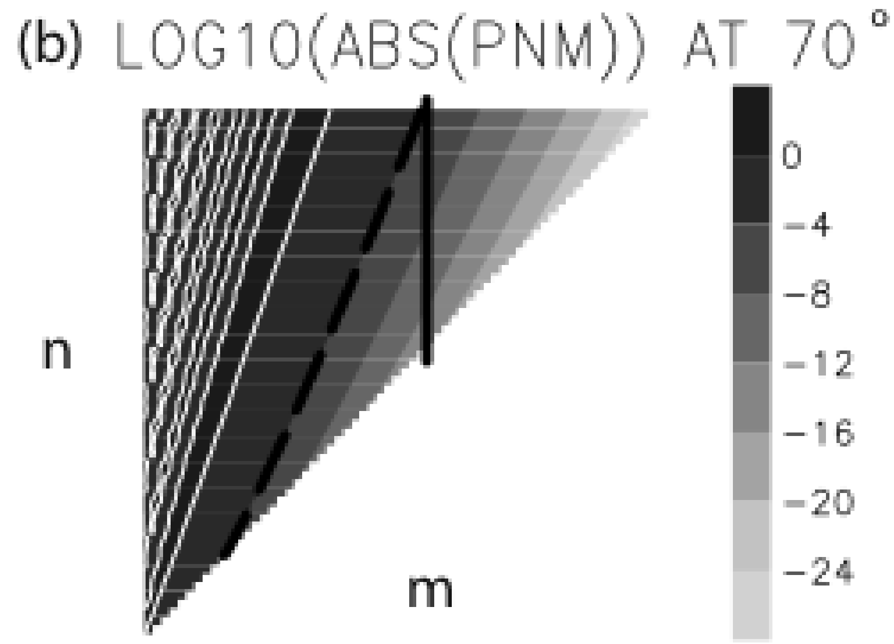


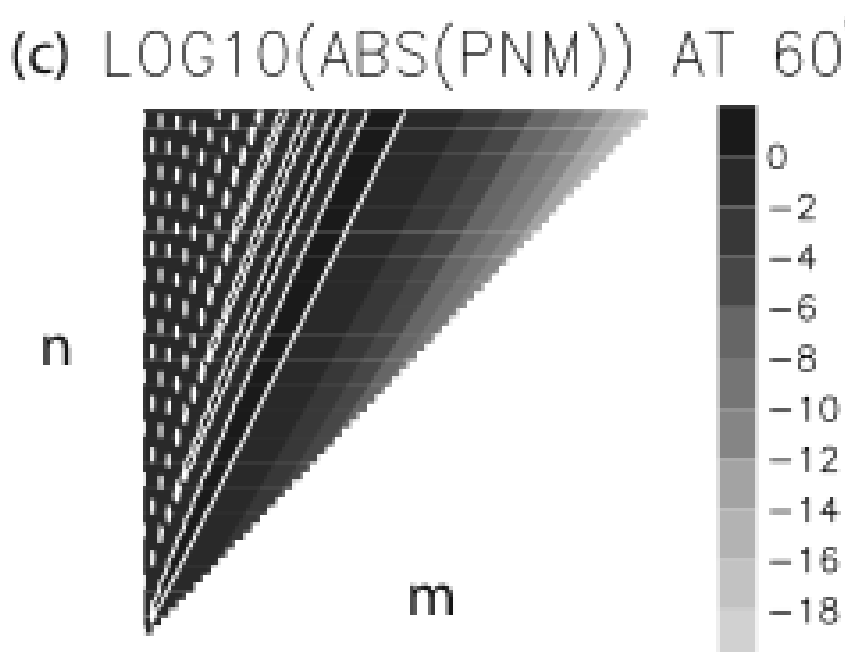

Psfc (pascal) dif(xf-non_xf) fcst 120 hr
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W
0
-800
-700
-600
-500
-400
-300
-200
-100
0
100
200
300
GrADS: COLA/IGES
2012-10-05-18:31

Fast Legendre Transform

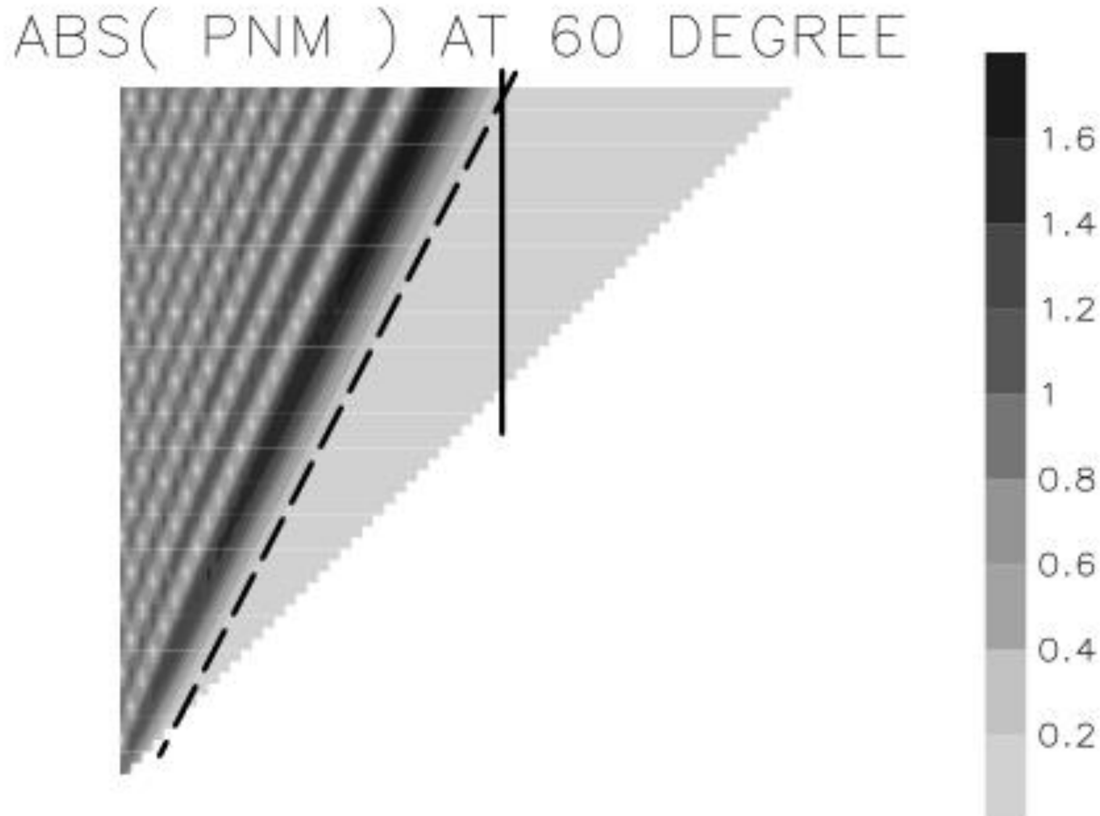
- ECMWF use butterfly method
  - M. Tygert (2010) J. Comp. Phys.
  - Based on interpolative decomposition
  - Reduced computation and accuracy
- Other FLTs
  - Examine P(n,m) again
  - Further reduce computation
  - Looking for periodic coefficients


(a) LOG10(ABS(PNM)) AT 80°
n
m
0
−4
−8
−12
−16
−20
−24
−28
−32
−36
−40
−44

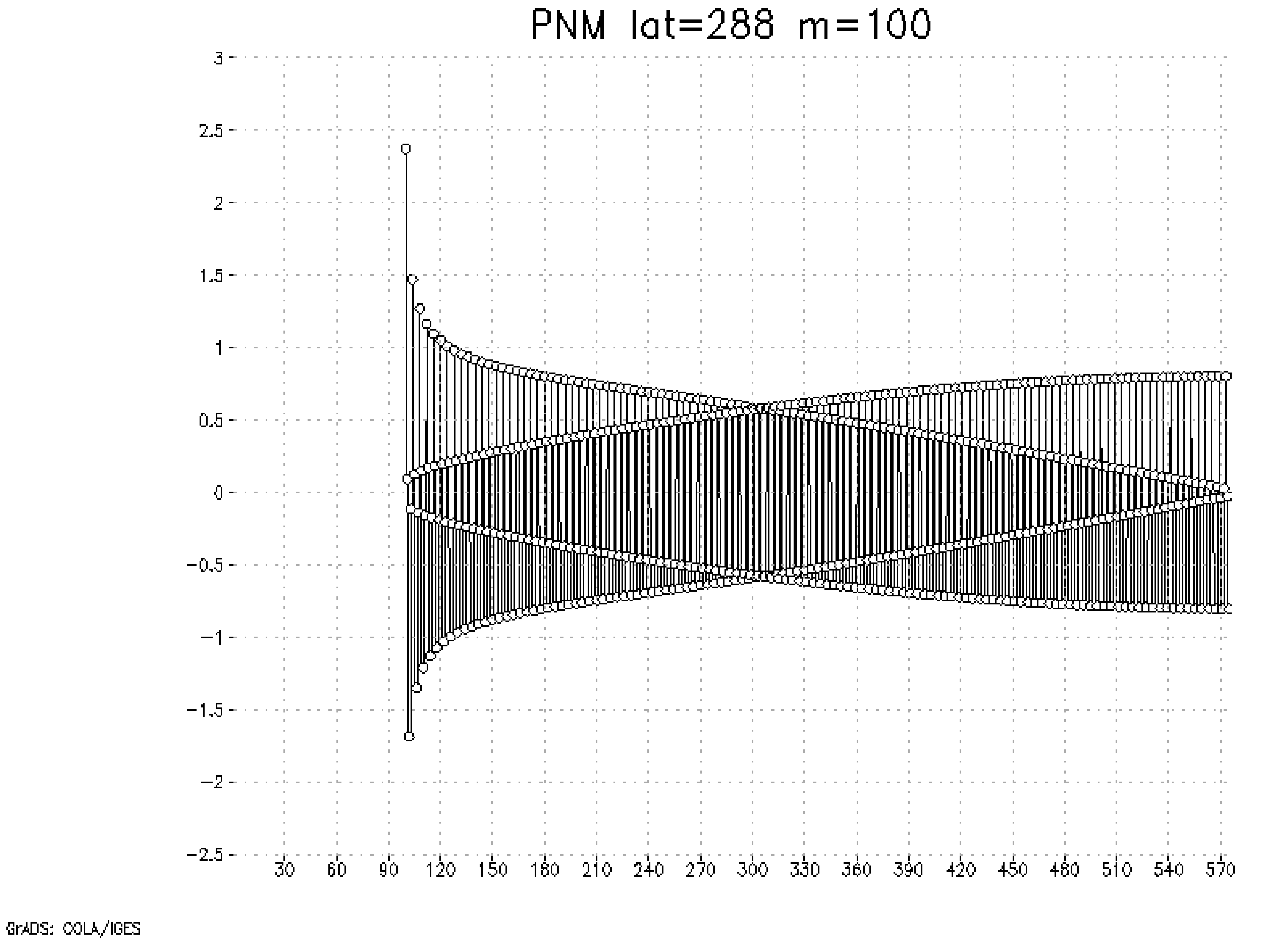

(b) LOG10(ABS(PNM)) AT 70°
n
m
0
−4
−8
−12
−16
−20
−24

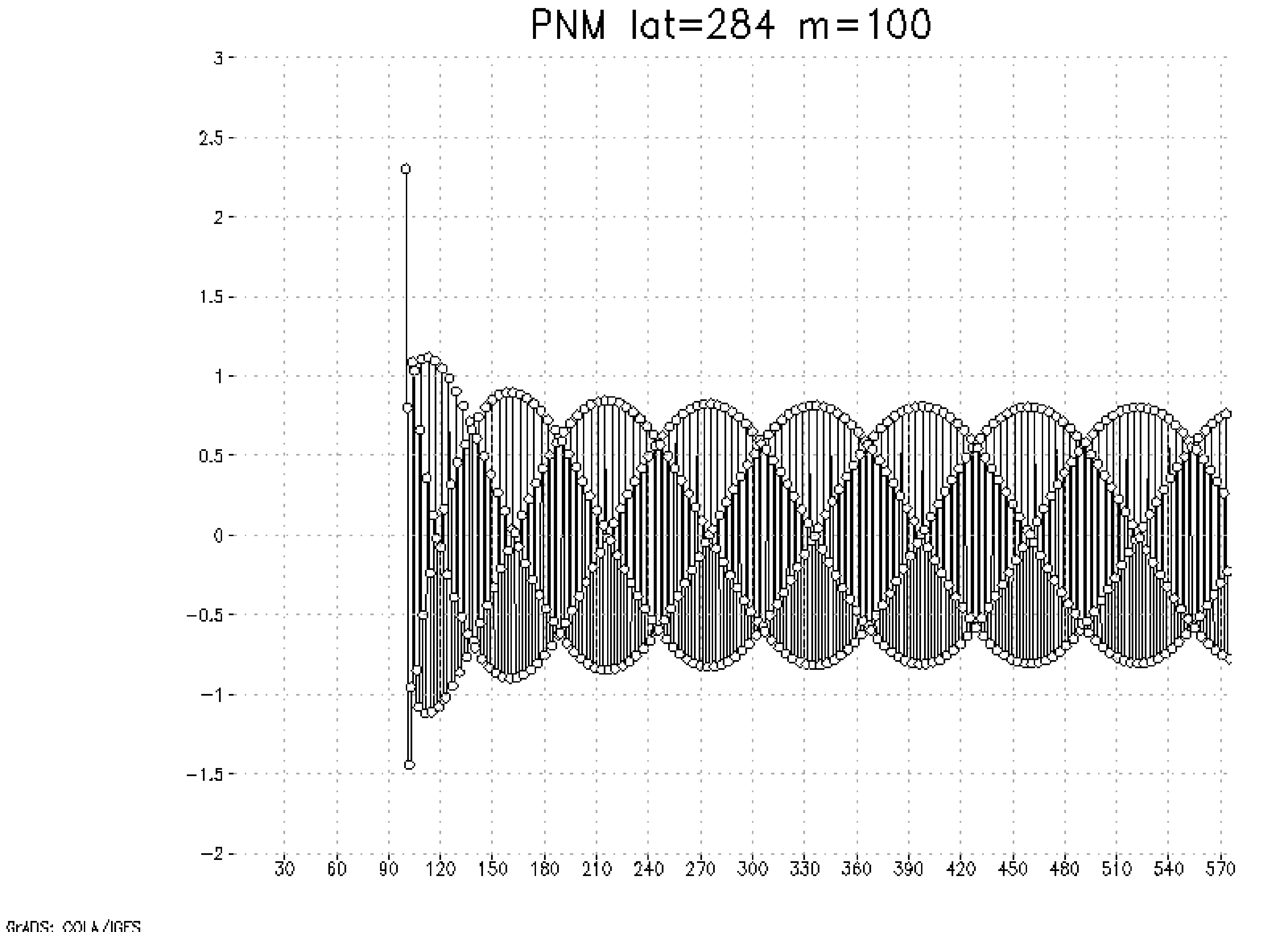

(c) LOG10(ABS(PNM)) AT 60°
n
m
0
−2
−4
−6
−8
−10
−12
−14
−16
−18

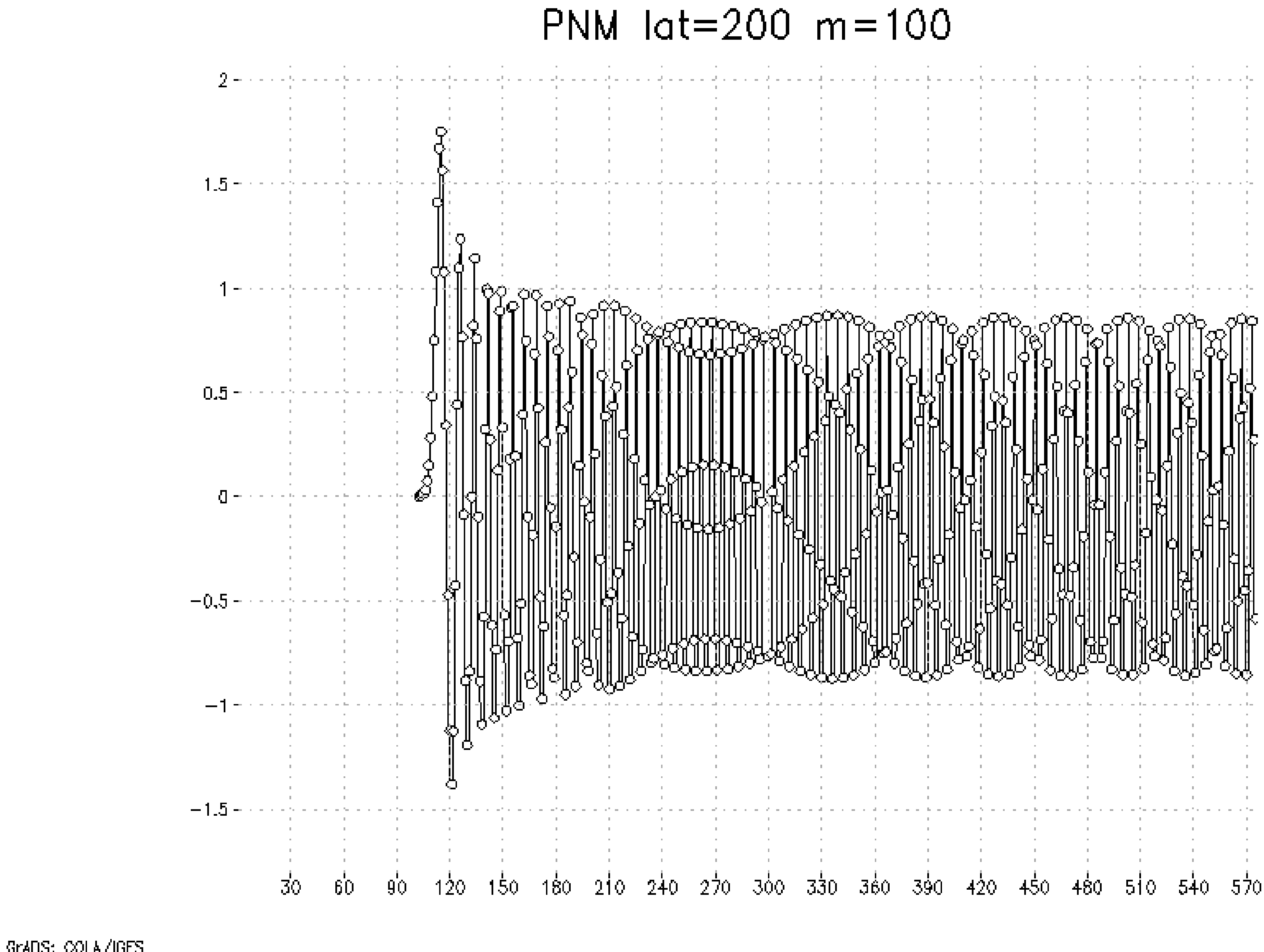
Juang 2004



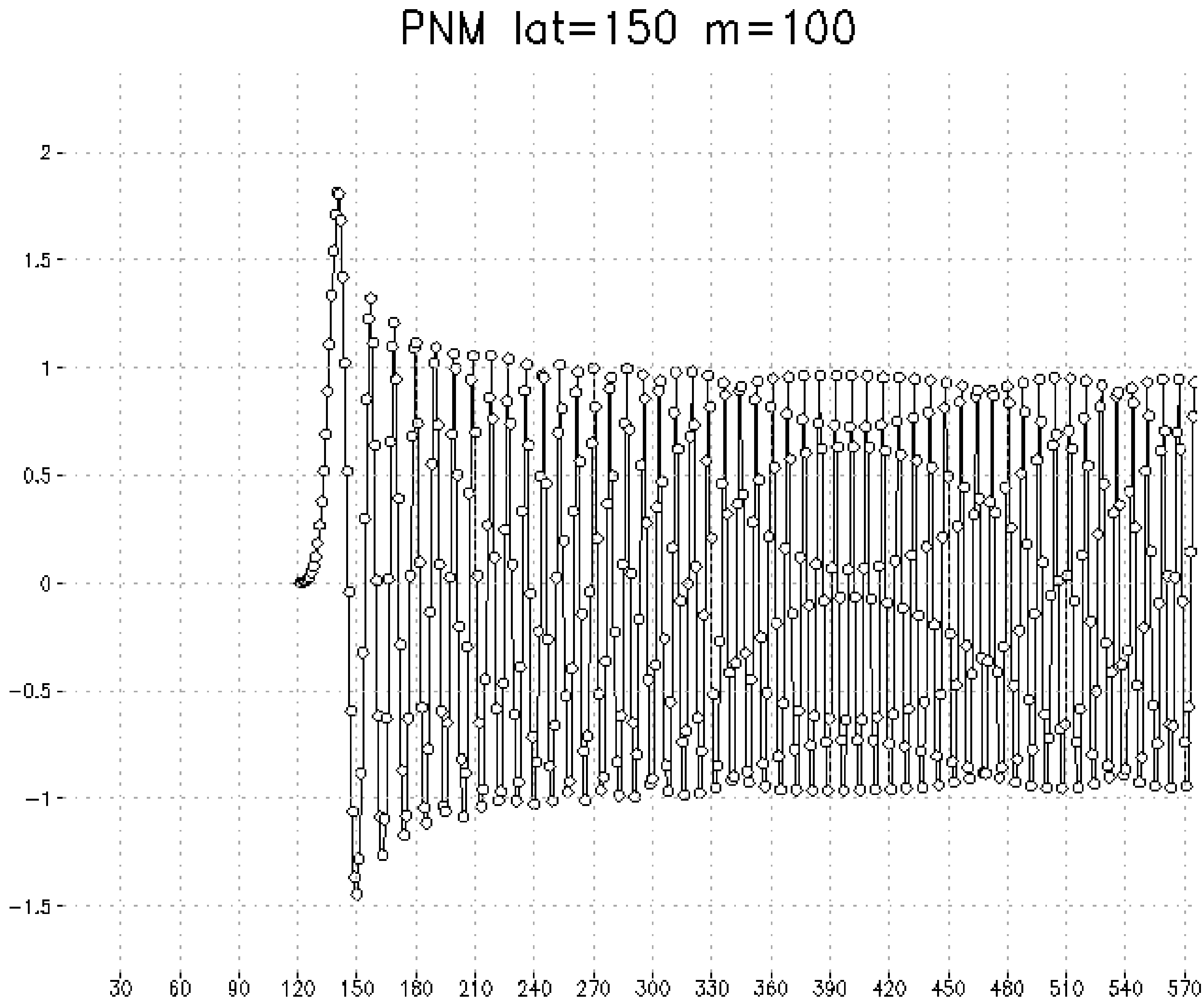
Some kinds of periodic and reduced computation due to accuracy


PNM lat=288 m=100
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
30 60 90 120 150 180 210 240 270 300 330 360 390 420 450 480 510 540 570
GrADS: COLA/IGES

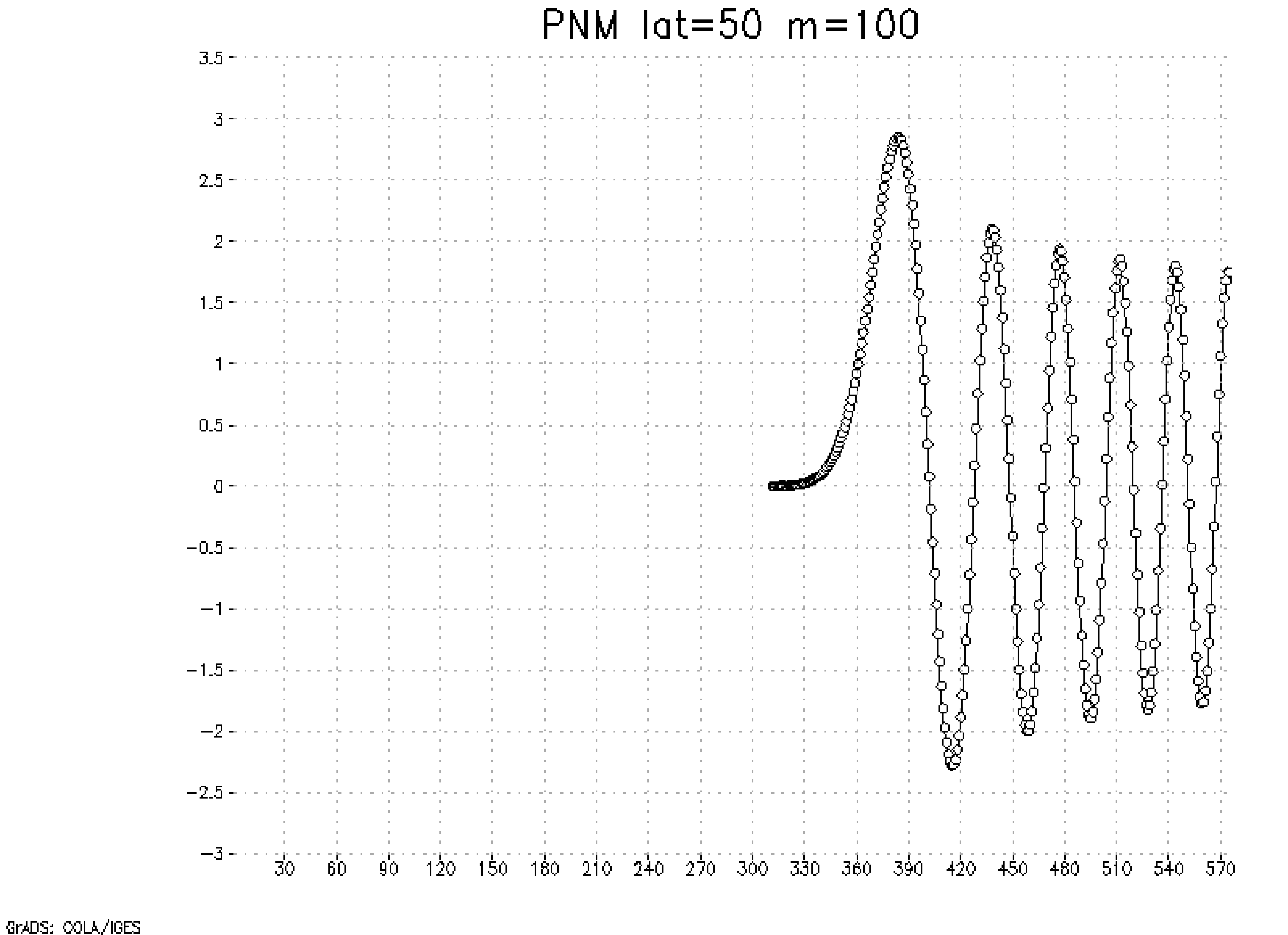

PNM lat=284 m=100
3
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
30 60 90 120 150 180 210 240 270 300 330 360 390 420 450 480 510 540 570
GrADS: COLA/IGES



PNM lat=200 m=100
2
1.5
1
0.5
0
-0.5
-1
-1.5
30
60
90
120
150
180
210
240
270
300
330
360
390
420
450
480
510
540
570
GrADS: COLA/IGES



GrADS: COLA/IGES


PNM lat=50 m=100
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
−3
30
60
90
120
150
180
210
240
270
300
330
360
390
420
450
480
510
540
570
GrADS: COLA/IGES

Any given lat and m, all Pnm are within a range

To do from spectral to grid, we do following

$$q^m = \sum_{n=m}^{N} q_n^m P_n^m$$

And it can be extended as

$$q^m = q_m^m P_m^m + q_{m+1}^m P_{m+1}^m + q_{m+2}^m P_{m+2}^m + \ldots\ldots + q_N^m P_N^m$$

which needs N-m+1 times of multiplication

If all of other Pnm are very close to the first three Pnm, then we can have

$$q^m \approx (q_m^m + \ldots.) P_m^m + (q_{m+1}^m + \ldots.) P_{m+1}^m + (q_{m+2}^m + \ldots.) P_{m+2}^m$$

which needs only three multiplications, saving N-m+1-3

T574 (1152x576) Total multiplication op is 37969901

RST numreduce=4	RLT	Accuracy	Total multiply opr saved	Percent saved
Yes			5632194	15%
Yes	Yes	0.001	10852398	28%
Yes	Yes	0.003	16765836	44%
Yes	Yes	0.005	20293820	53%

T1148 (2304x1152) Total multiplication op is 299673620

RST numreduce=4	RLT	Accuracy	Total multiply opr saved	Percent saved
Yes			47521344	15%
Yes	Yes	0.001	114898710	38%
Yes	Yes	0.003	172002317	57%
Yes	Yes	0.005	199849743	67%

Conclusion

- We are ready to do fine resolution global spectral model.
- We have made spectral transform valid in fine resolution.
- We have a method to speedup transform, have not decided between fast and reduced Legendre transform